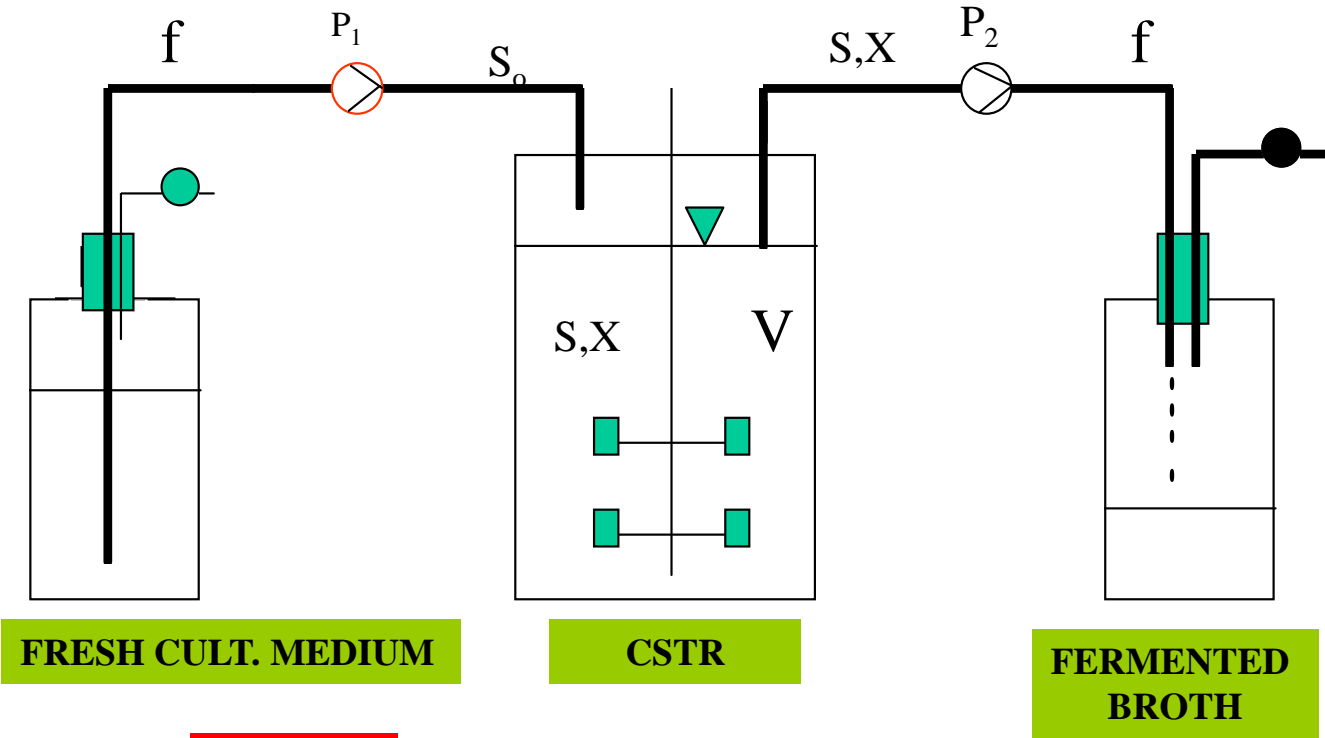


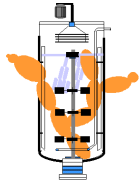
# CONTINUOUS FERMENTATION



Cell mass: 
$$V \frac{dx}{dt} = V \left( \frac{dx}{dt} \right)_{\text{growth}} - f \cdot x$$

$i^{\text{th}}$  substrate: 
$$V \frac{dS_i}{dt} = fS_{i,0} - fS_i - V \frac{1}{Y_{x/S_i}} \left( \frac{dx}{dt} \right)_{\text{growth}}$$

$$\frac{f}{V} = D$$



## CONTINUOUS FERMENTATION

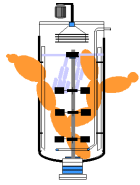
$$\frac{f}{V} = D$$

$\mathbf{m^3/h}$   
 $\mathbf{m^3}$

$\mathbf{h^{-1}}$  Higítási sebesség  
Dilution rate

$$\frac{1}{D} = \bar{t}$$

$\mathbf{h}$  Átlagos tartózkodási idő  
Mean residence time



## CONTINUOUS FERMENTATION

In the case of one limiting S ( if MONOD model holds ):

$$\frac{dx}{dt} = \mu x - Dx = (\mu - D)x = \left( \mu_{\max} \frac{S}{K_S + S} - D \right) x$$

$$\frac{dS}{dt} = D(S_0 - S) - \frac{\mu x}{Y}$$

In steady state

$$\frac{dx}{dt} = 0 \quad \text{and} \quad \frac{dS}{dt} = 0$$

$$D = \mu_{\max} \frac{S}{K_S + S} \quad \Rightarrow \quad \bar{S} = \frac{K_S D}{\mu_{\max} - D}$$

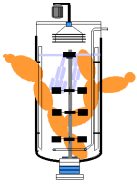
$$D(S_0 - \bar{S}) = \frac{\mu x}{Y}$$

$$\bar{x} = Y(S_0 - \bar{S}) = Y \left( S_0 - \frac{K_S D}{\mu_{\max} - D} \right)$$

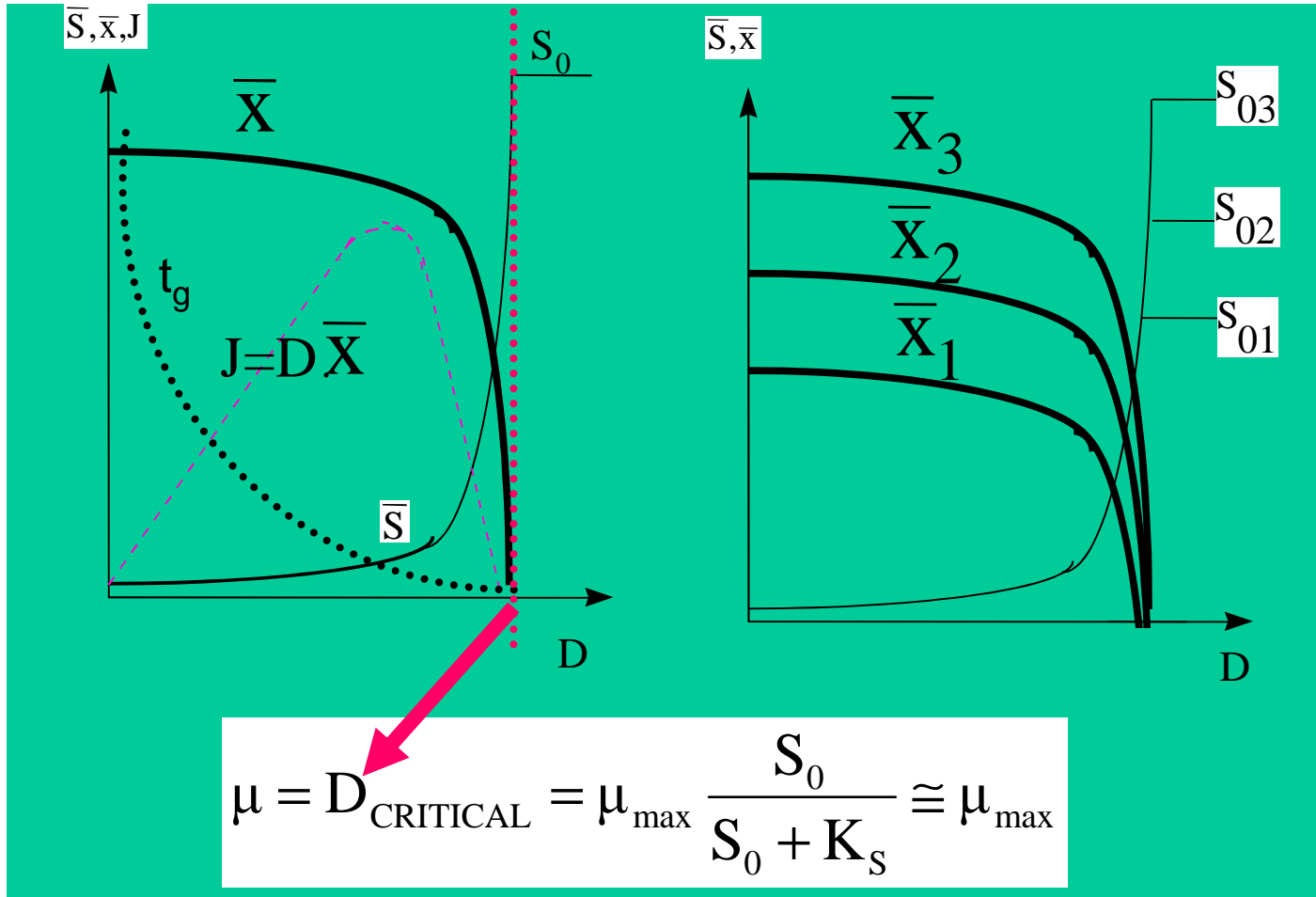
Necessary and  
enough condition of the  
steady state

$$\mu = D$$

**CHEMOSTAT**

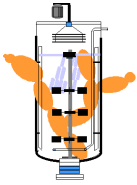


# CONTINUOUS FERMENTATION



(corresponds to the declining phase)





## CONTINUOUS FERMENTATION

$$J = D \cdot x \quad [\text{g/l.h}] \quad \text{or} \quad [\text{kg/m}^3\text{h}]$$

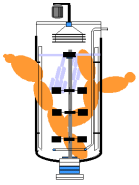
$$J = D \cdot \bar{x} = D \cdot Y \left( S_0 - \frac{K_S D}{\mu_{\max} - D} \right) = \text{max!!!}$$

$$\frac{\partial J}{\partial D} = 0 \quad \longrightarrow \quad D_{\max} = \mu_{\max} \left( 1 - \left( \frac{K_S}{S_0 + K_S} \right)^{1/2} \right)$$

$$\bar{x}_{\max} = Y \left[ S_0 + K_S - \sqrt{K_S (S_0 + K_S)} \right]$$

$$\text{○} = D_{\max} \bar{x}_{\max} = \mu_{\max} Y \left[ 1 - \left( \frac{K_S}{K_S + S_0} \right)^{1/2} \right] \cdot \left[ K_S + S_0 - \sqrt{K_S (S_0 + K_S)} \right] =$$

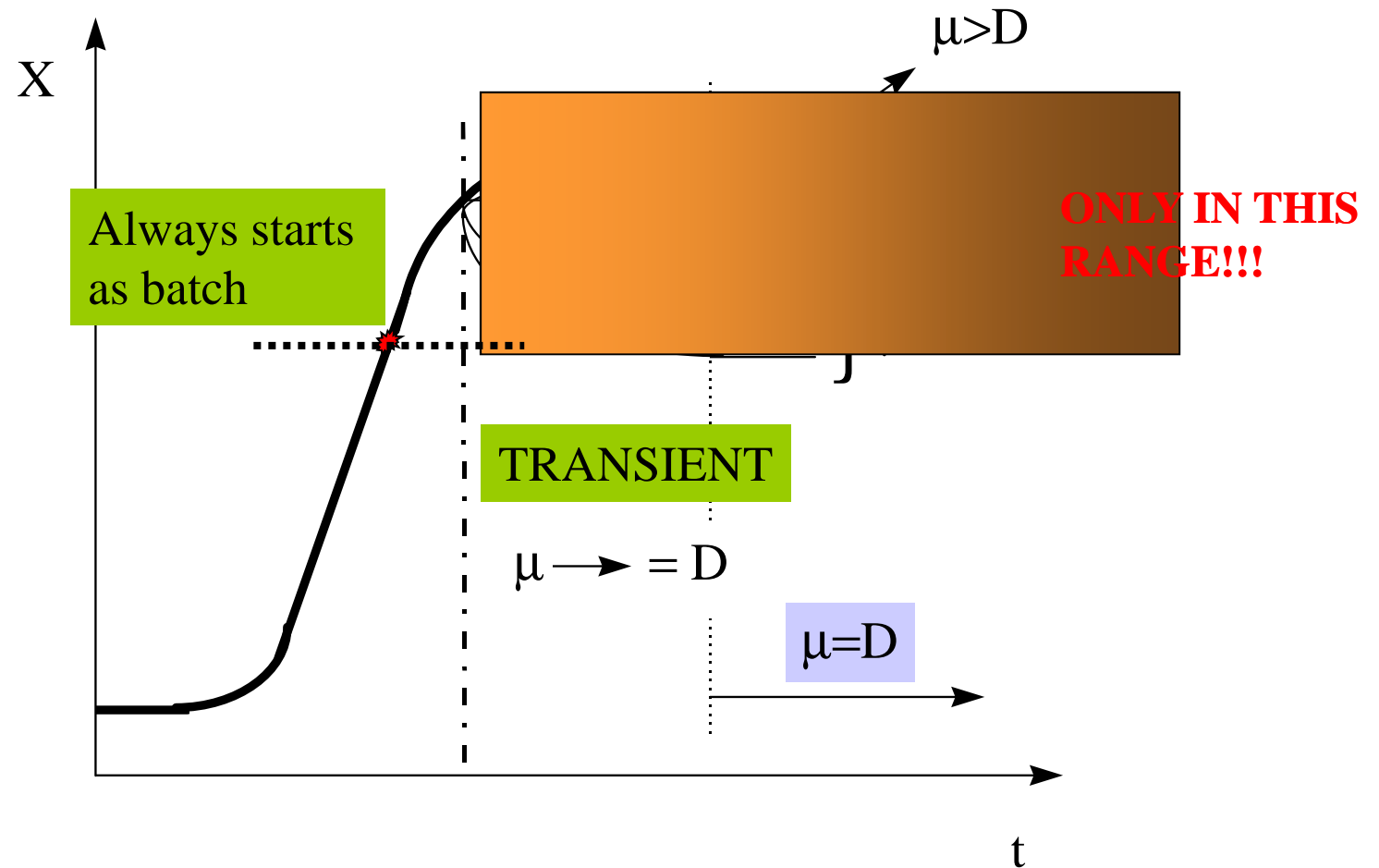
$$= Y \mu_{\max} S_0 \left( \sqrt{\frac{K_S + S_0}{S_0}} - \sqrt{\frac{K_S}{S_0}} \right)^2 \text{○}$$



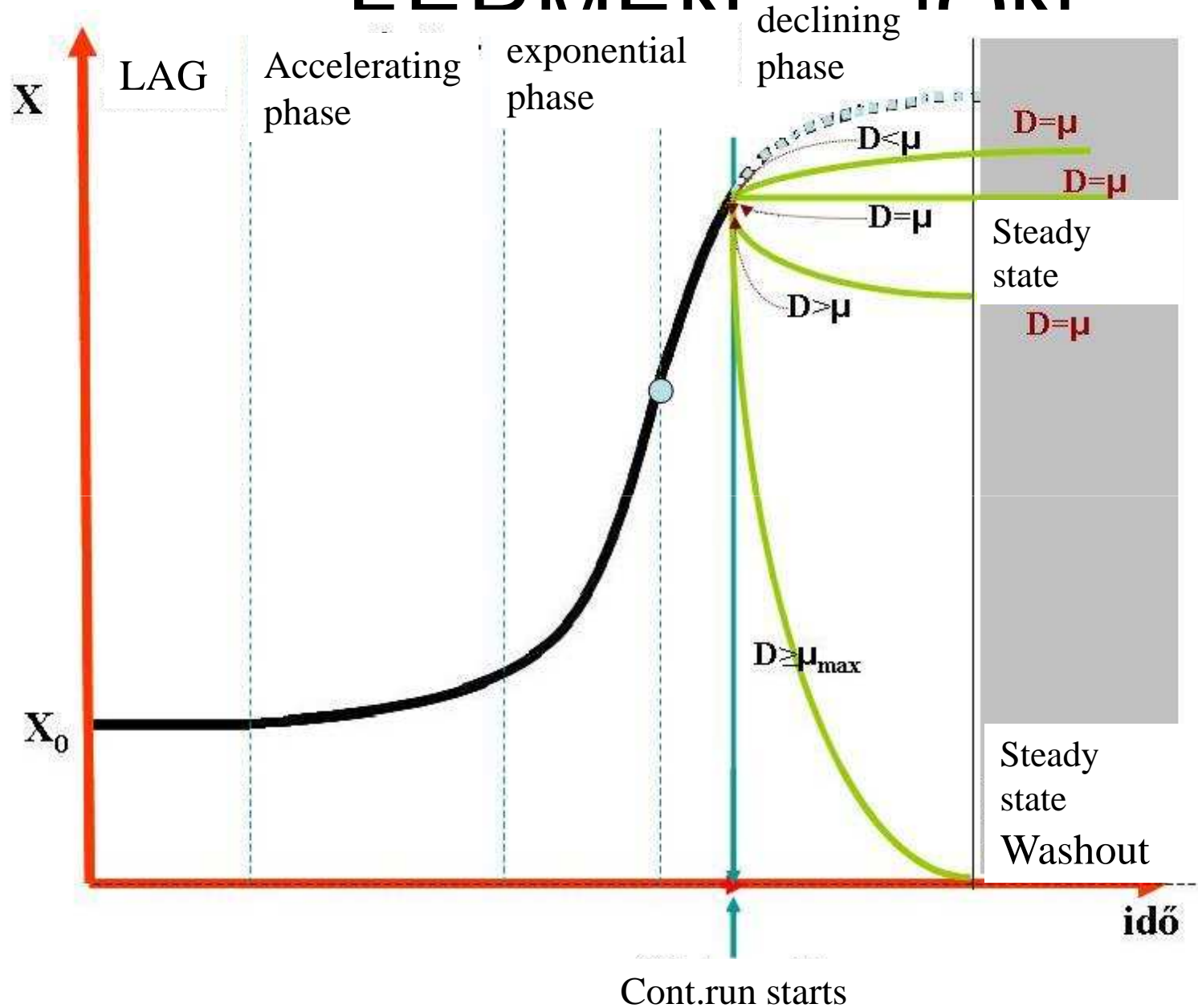
# CONTINUOUS FERMENTATION

## Transient behaviour

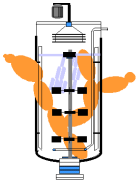
1. After start: transient from batch to continuous operation



# CONTINUOUS SEDIMENTATION





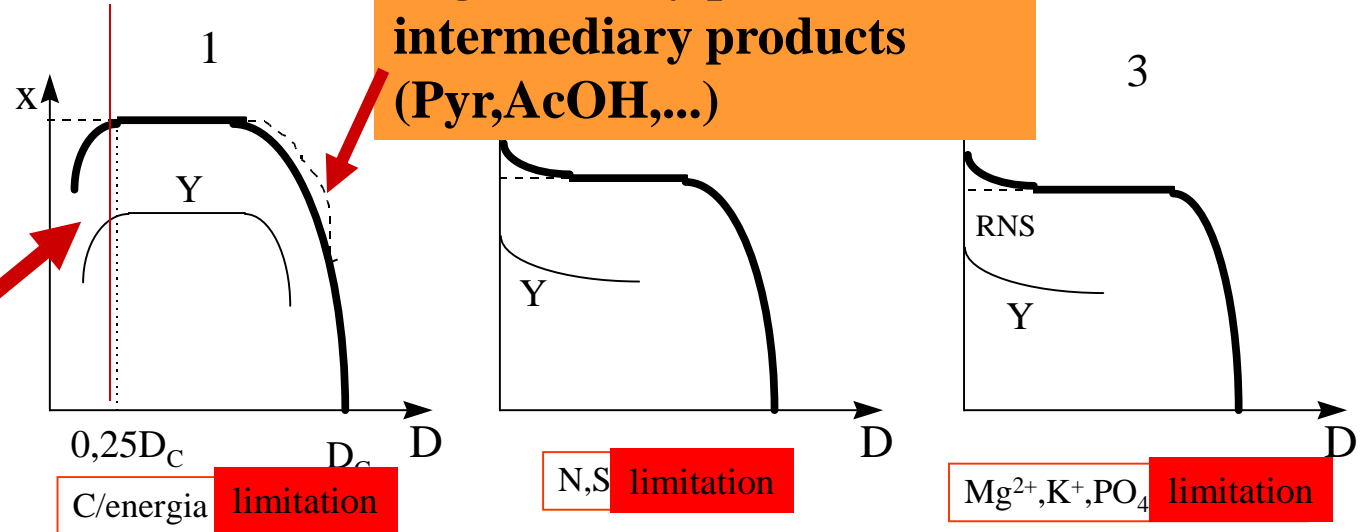


# CONTINUOUS FERMENTATION

## Alterations from ideal behaviour

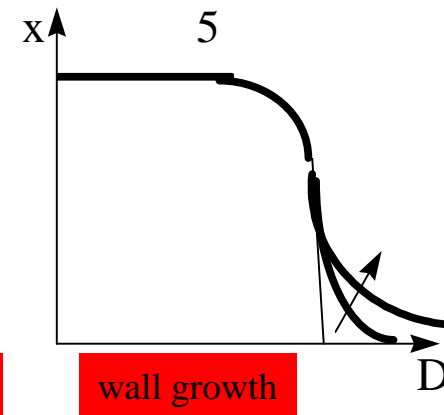
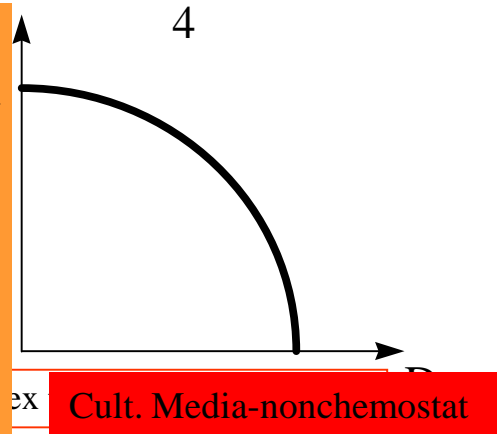
High velocity production of intermediary products (Pyr, AcOH, ...)

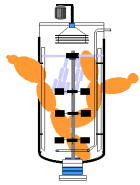
$D < 0,25D_C$



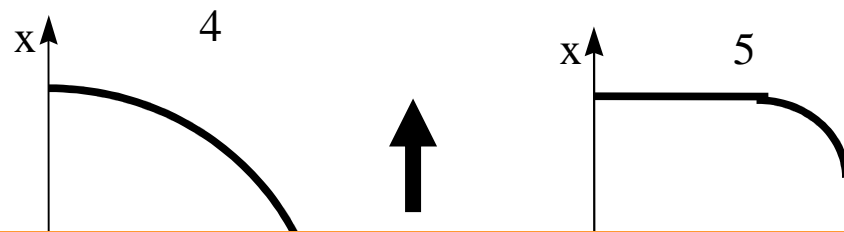
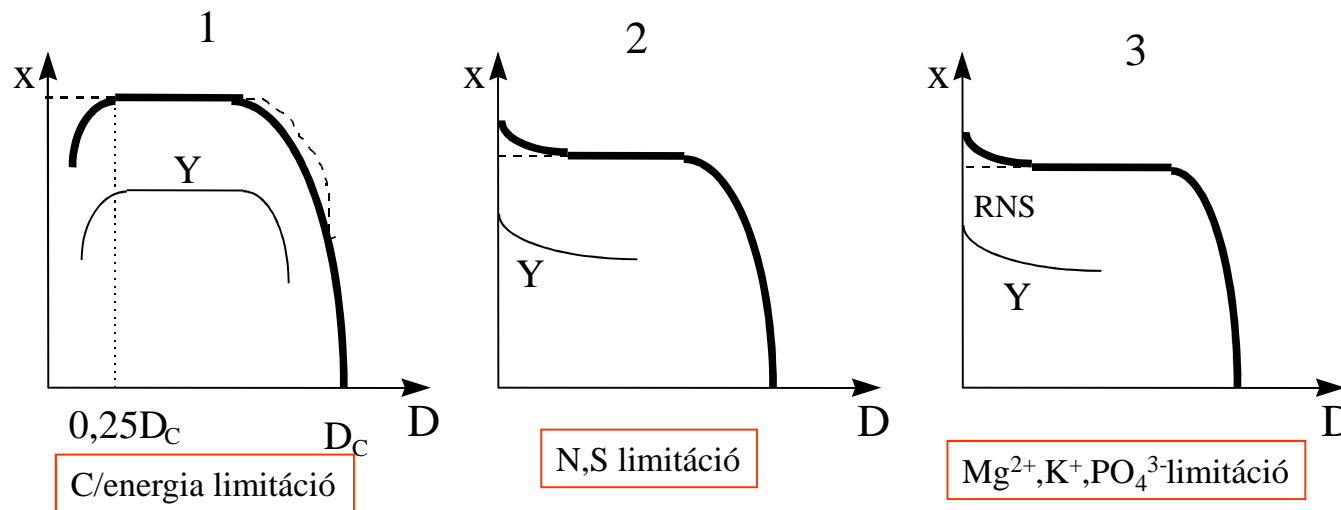
$$\frac{dS}{dt} = D(S_0 - S) - \left( \frac{1}{Y_C} - \frac{1}{Y_{EG}} - \frac{m}{\mu} \right) \mu x$$

$$x = \frac{\left( S_0 - \frac{K_S D}{\mu_{max} - D} \right)}{\left( \frac{1}{Y_C} - \frac{1}{Y_{EG}} - \frac{m}{\mu} \right)}$$

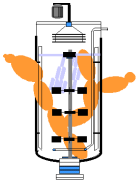




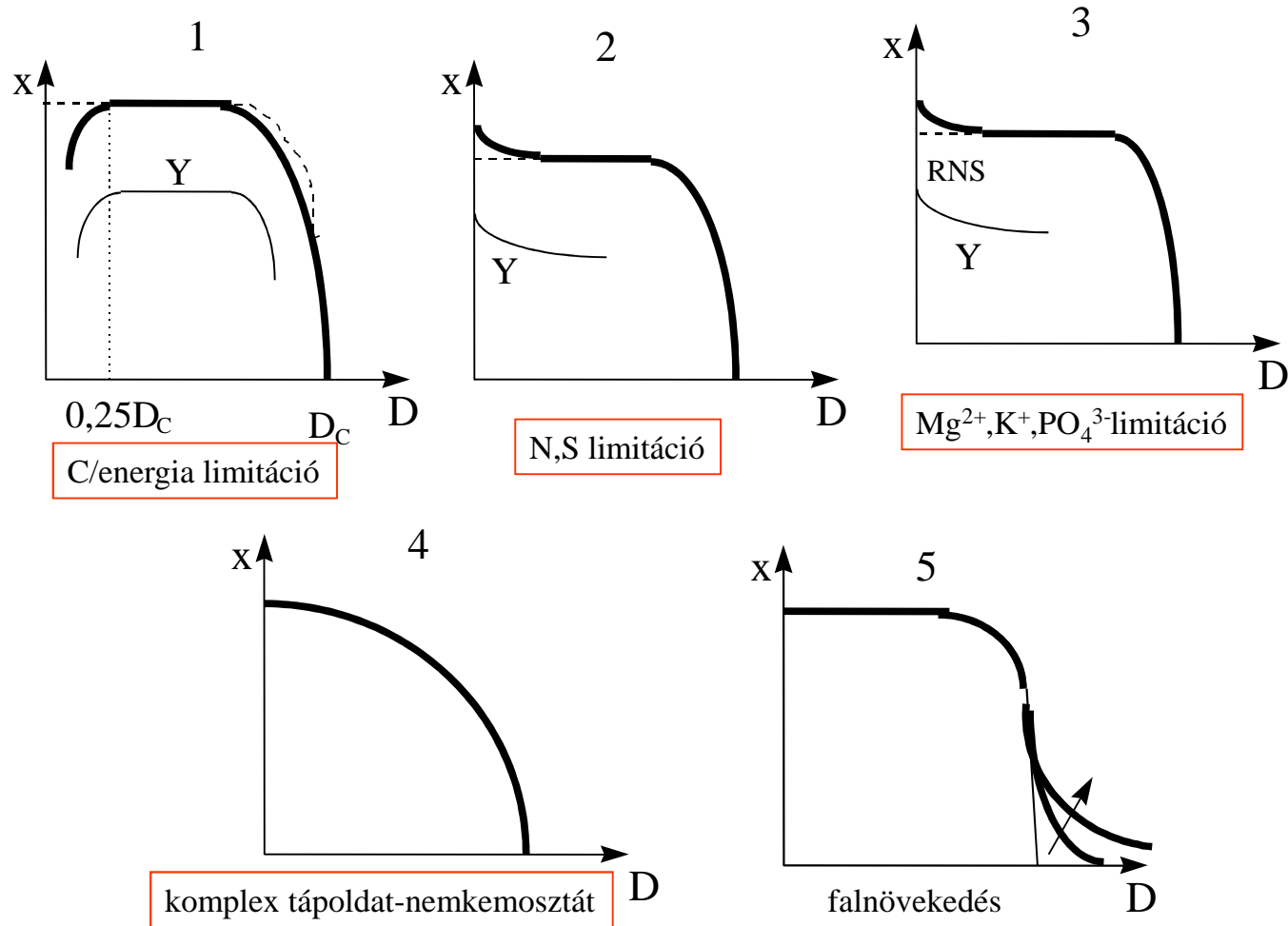
# CONTINUOUS FERMENTATION

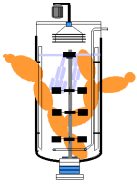


**N-forrás, vagy a kénforrás a limitáló tényező**  
**Kisebb  $D$ -nél a C/en forrás feleslegben van:**  
**Tartaléktápanyagok szintézise**  
**(poliszaharidok, lipidek,  $\beta$ -OH-butirát)**

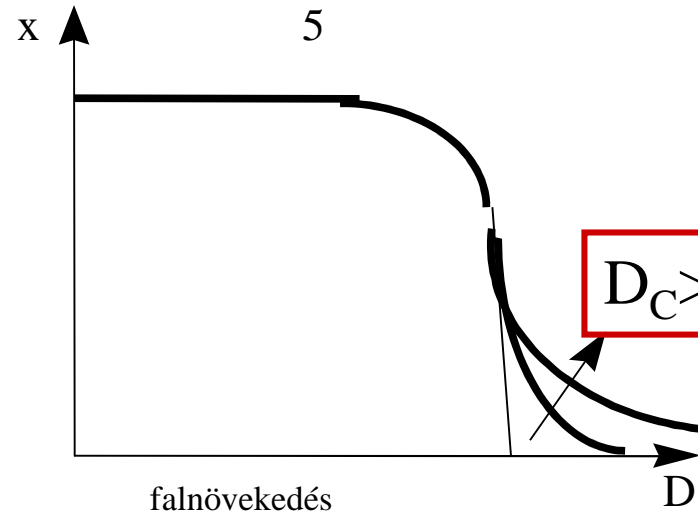


# CONTINUOUS FERMENTATION





# CONTINUOUS FERMENTATION

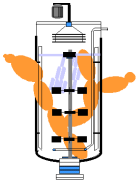


$$D\bar{x} = \mu\bar{x} + \mu\bar{x}_f$$

$$D(S_0 - \bar{S}) = (\mu\bar{x} + \mu\bar{x}_f) / Y_{x/S}$$

$$\bar{x} = Y_{x/S}(S_0 - \bar{S})$$

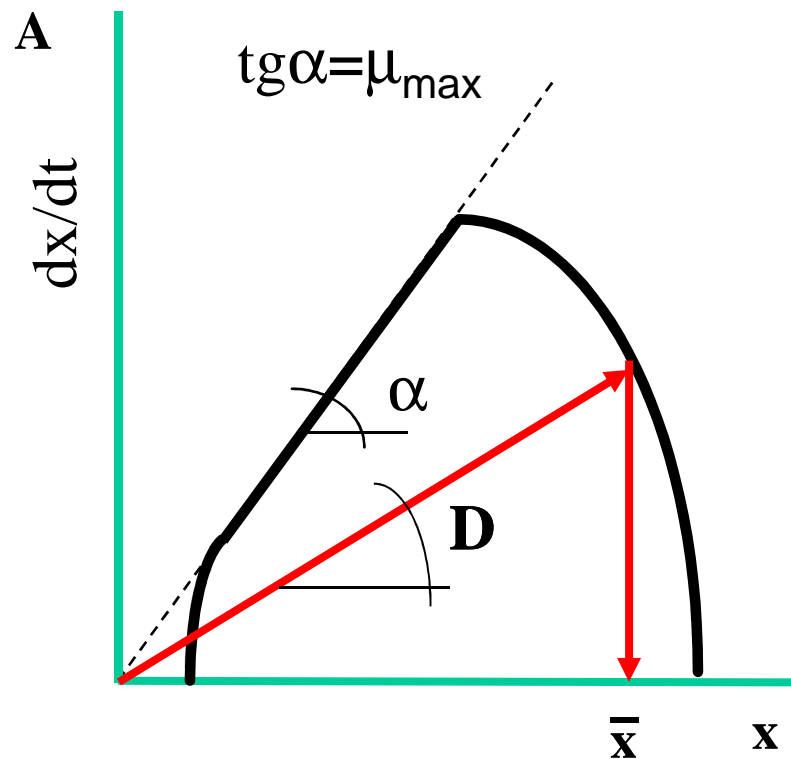
$$D = \mu \left( 1 + \frac{\bar{x}_f}{\bar{x}} \right)$$



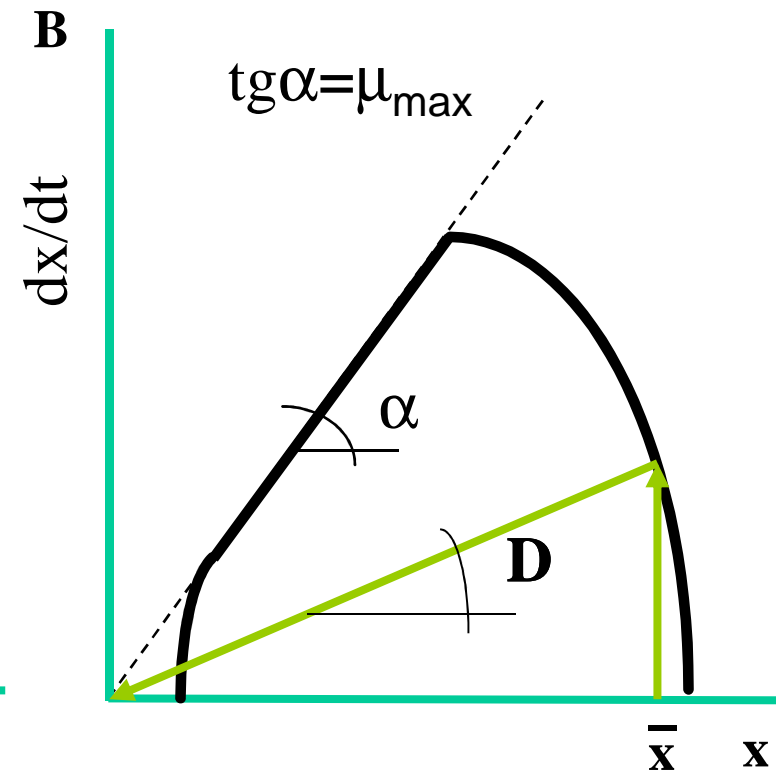
# CONTINUOUS FERMENTATION

## Design of the chemostat

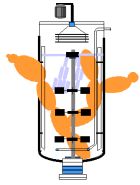
1. Known batch kinetics:  $\mu_{\max}$ ,  $Y$ ,  $K_S$ ,  $D$
2. Known batch growth curve (and derivative)



choose  $D$ - $t$ , what is  $x$ ?



Choose  $x$ ,  
What is the necessary  $D$ ?



# CONTINUOUS FERMENTATION

## Problems

**Volume control**

**aeration, foaming**

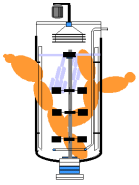
## USE OF CHEMOSTAT?

**ADVANTAGES: higher productivity  
balanced, limited growth  
measurement and control**

**SCP, bakers yeast, fodder yeast, (cell mass), primary metabolites:  
alcohol, beer**

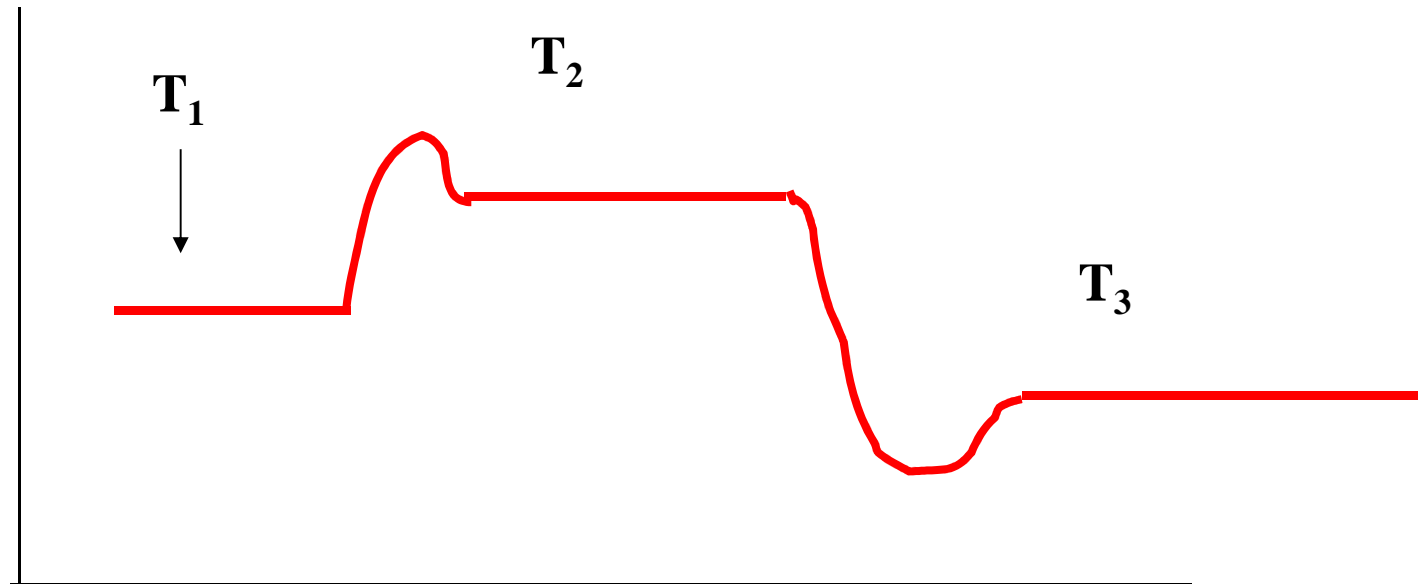
**research: kinetics, optimization,**

**but: secondary no, though penicillin...in lab scale**



# CONTINUOUS FERMENTATION

## OPTIMIZATION



**T: TEMPERATURE**  
**CULTURE MEDIA...**

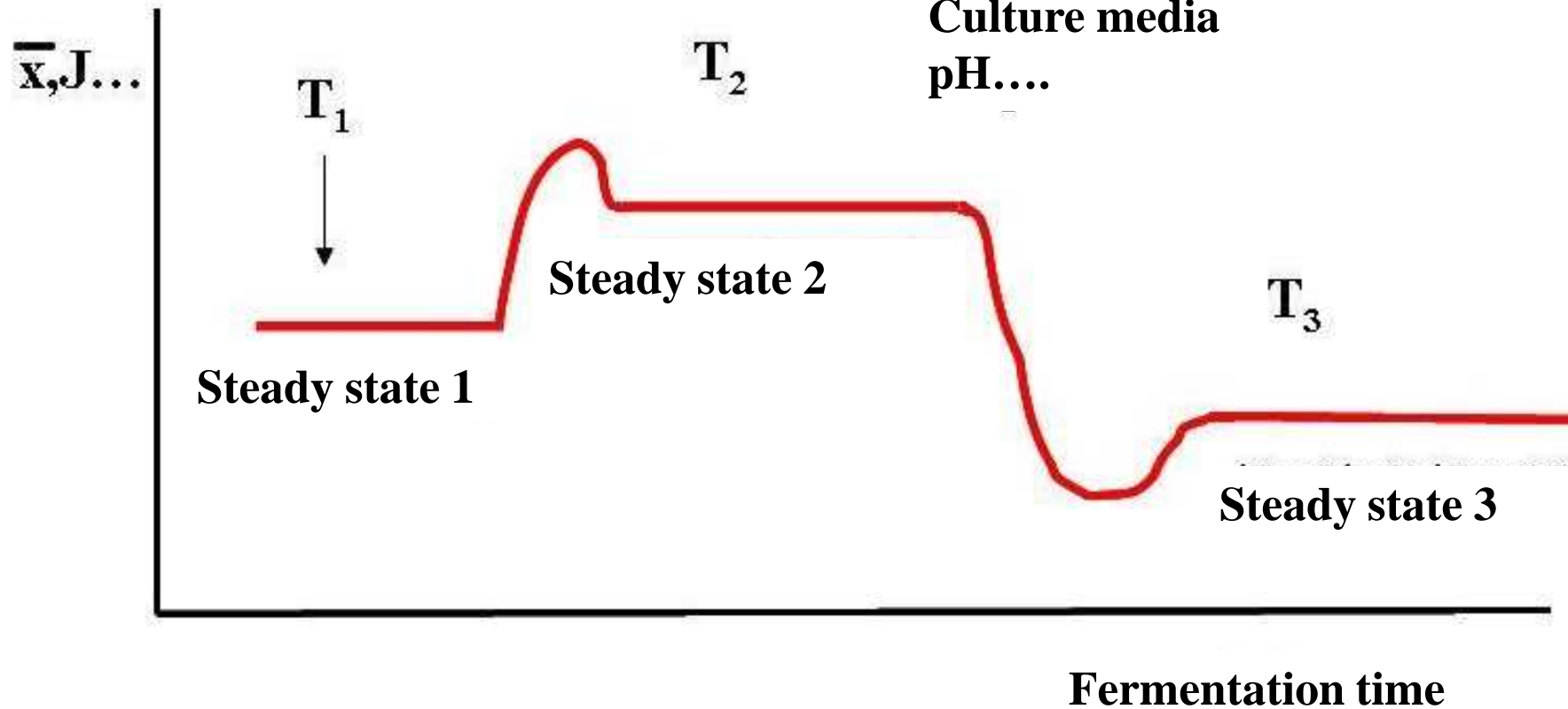
# CONTINUOUS FERMENTATION

## CONTINUOUS FERMENTATION

**T:** temperature

**Culture media**

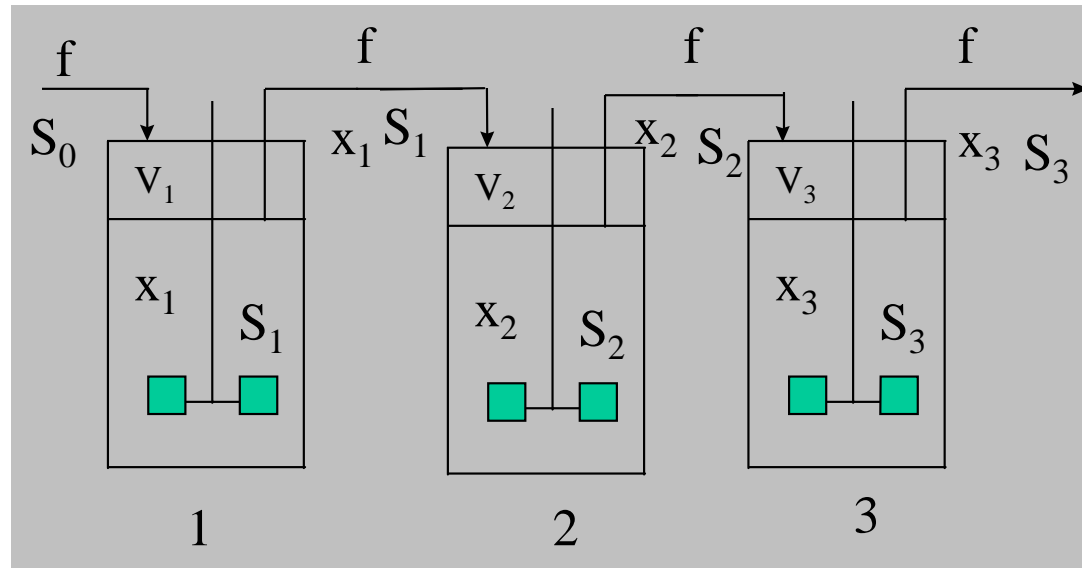
**pH....**



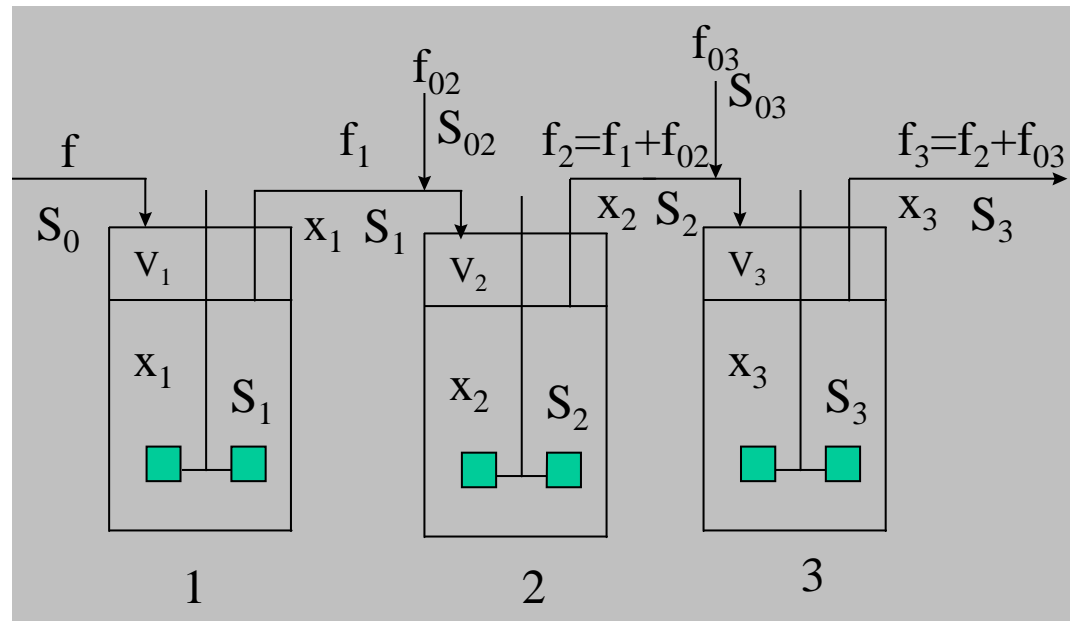


# CONTINUOUS FERMENTATION

**One stream,  
multiple stage**

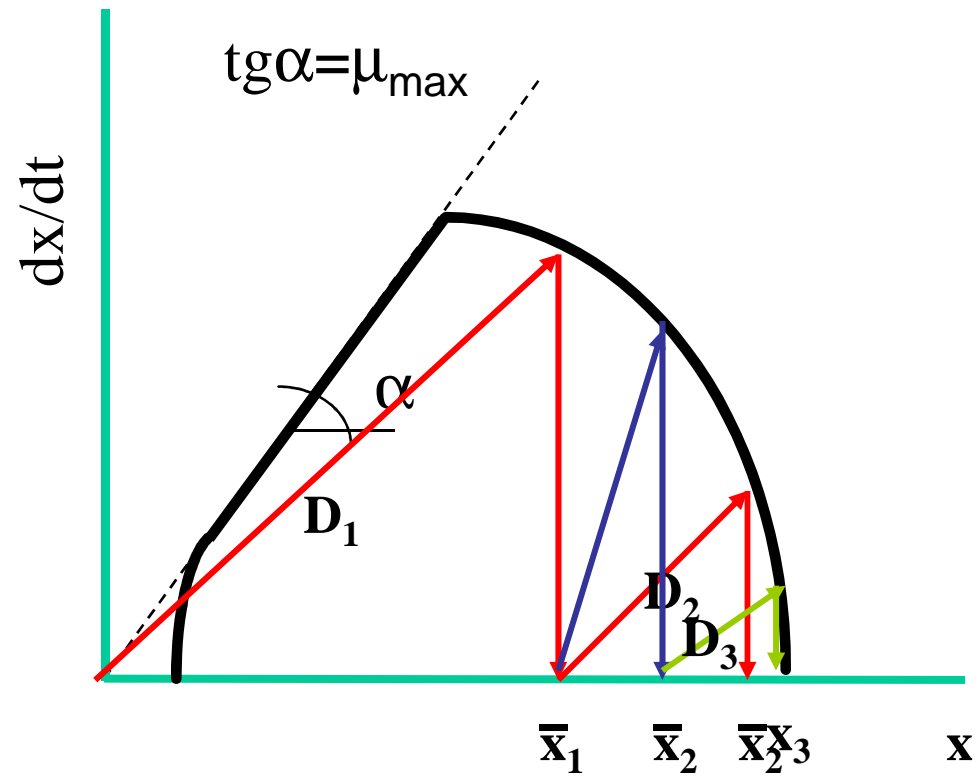


**Multiple stream,  
Multiple stage**



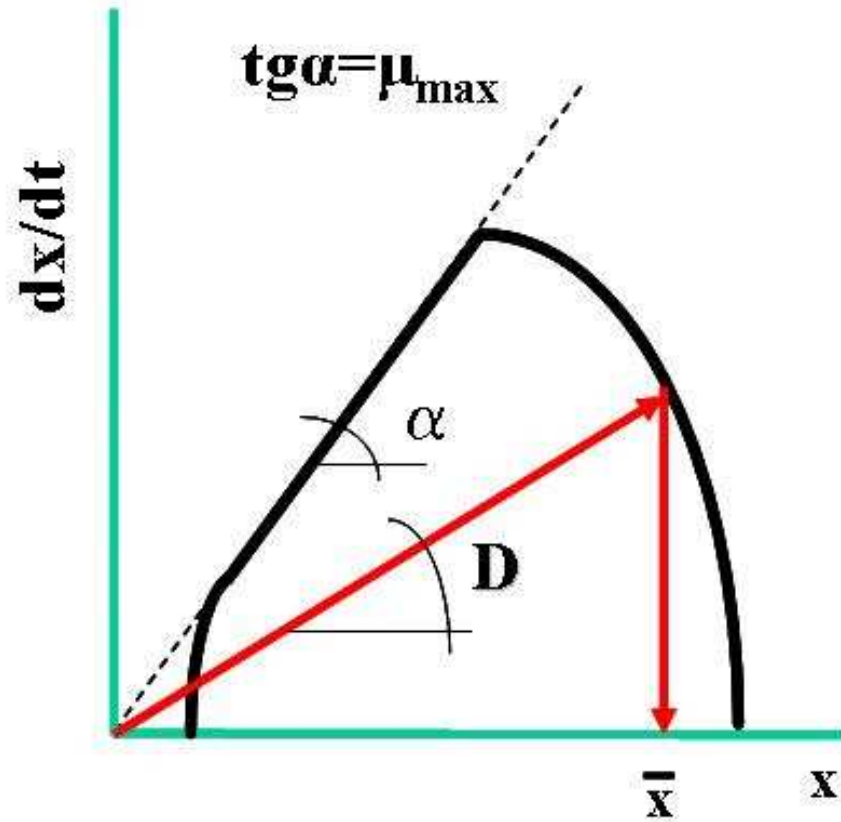
# CONTINUOUS FERMENTATION

design:



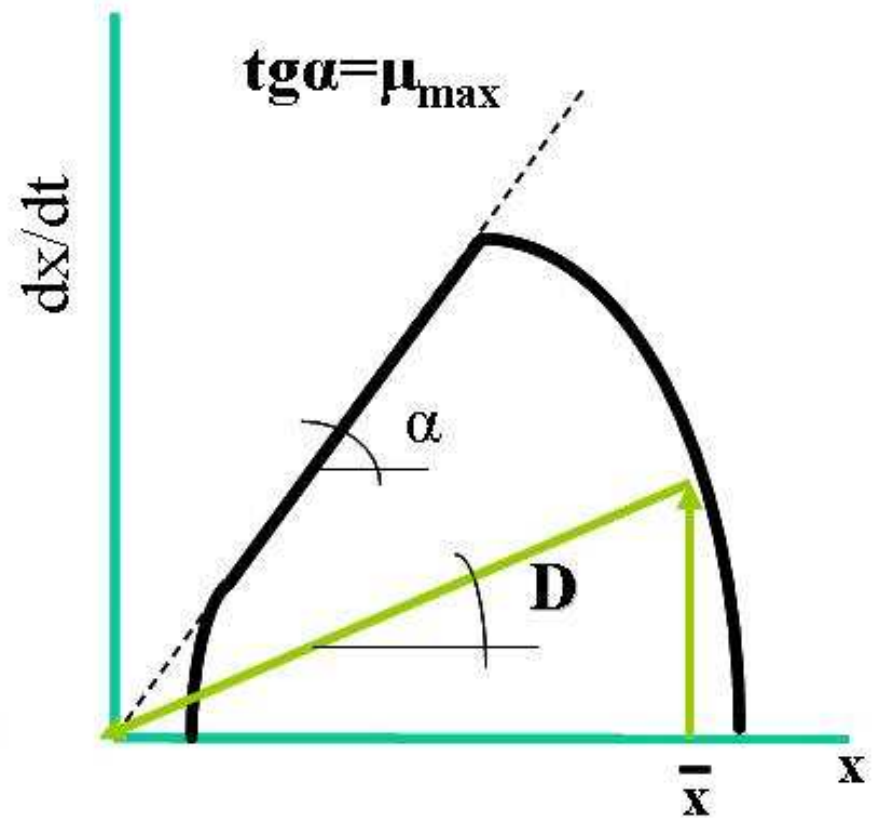
# CONTINUOUS FERMENTATION

A

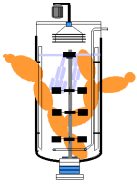


Choosing  $D$   
What the outlet will be?

B

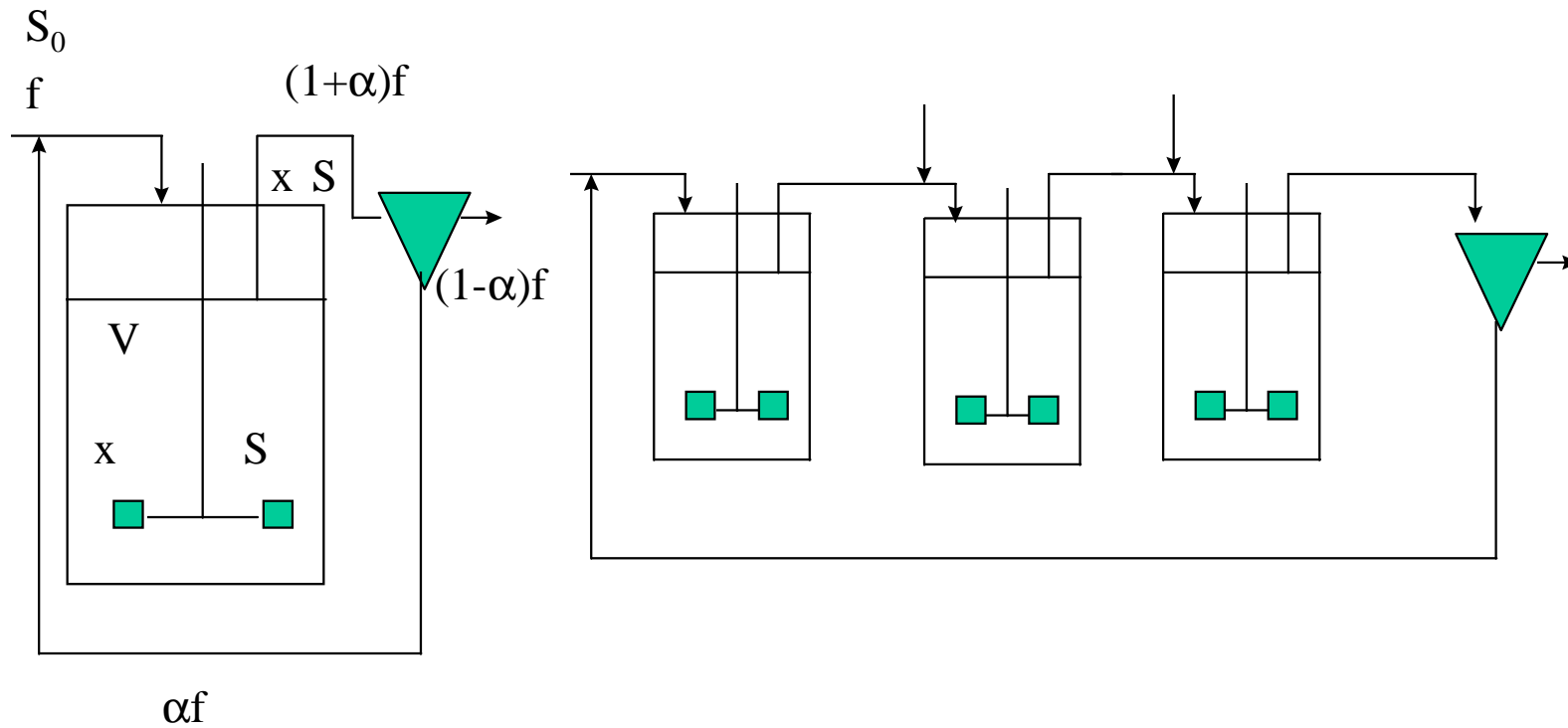


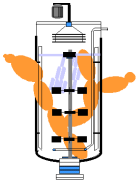
Choosing outlet  
What the  $D$  will be?



# CONTINUOUS FERMENTATION

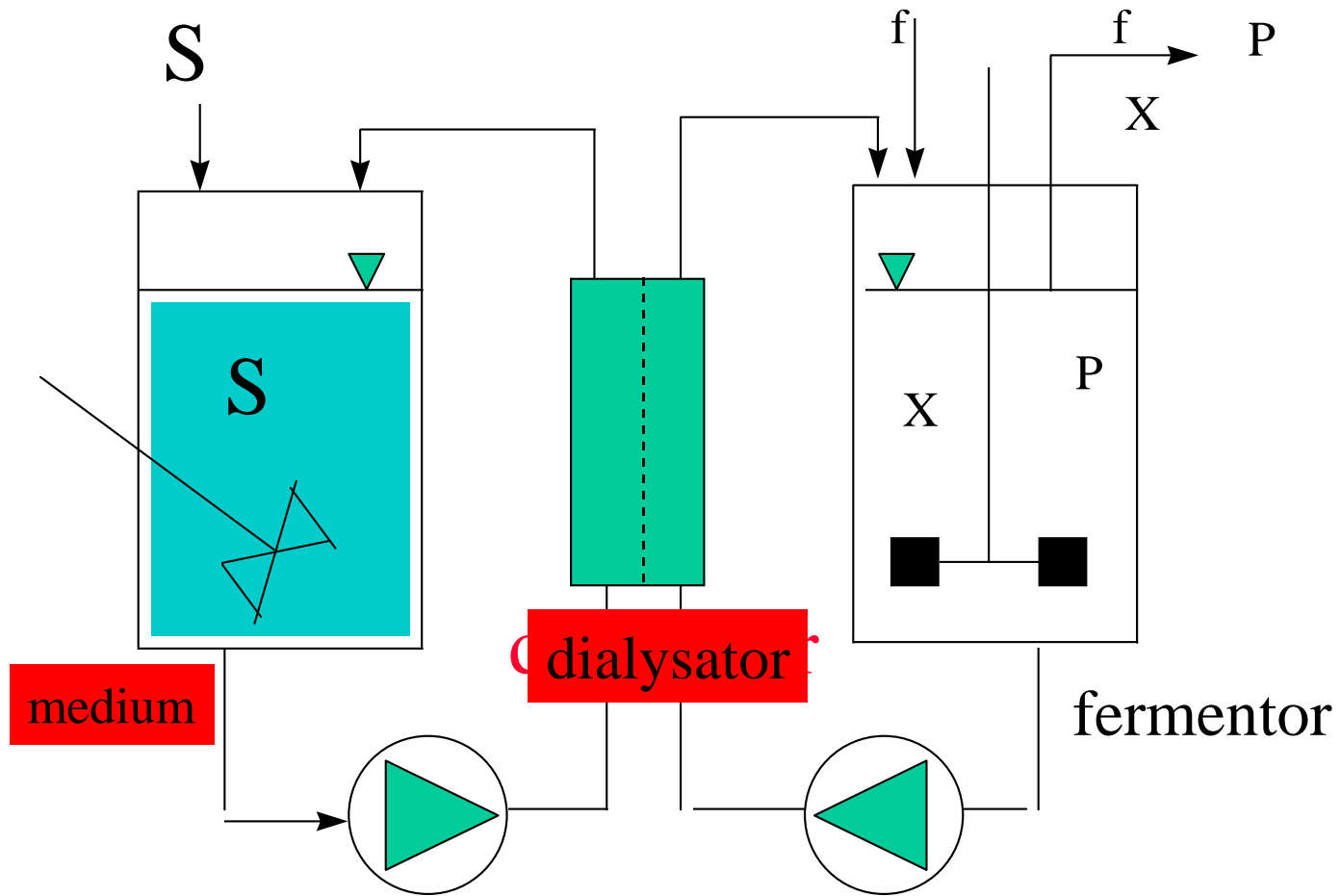
## Chemostats with recycle

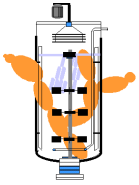




# CONTINUOUS FERMENTATION

## Special chemostat: dialysis culture

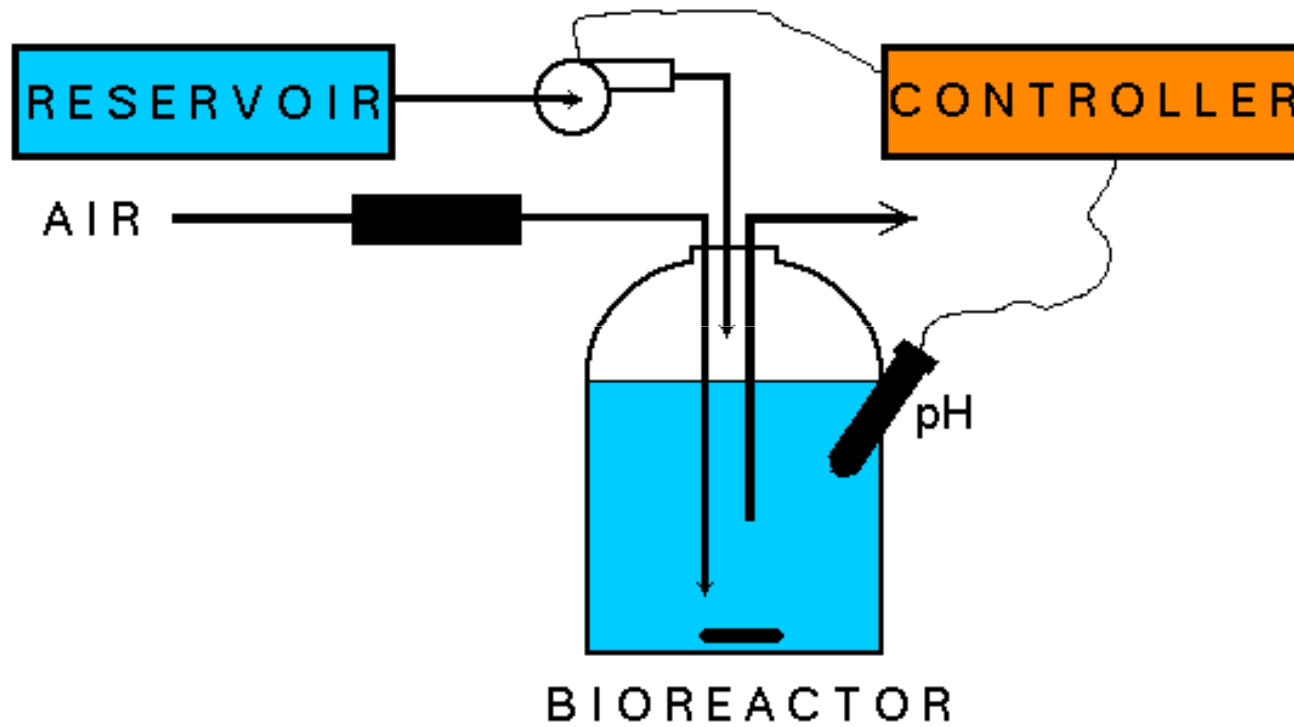


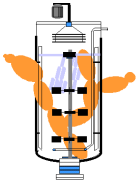


# CONTINUOUS FERMENTATION

## Auxostats

### pH-auxostat





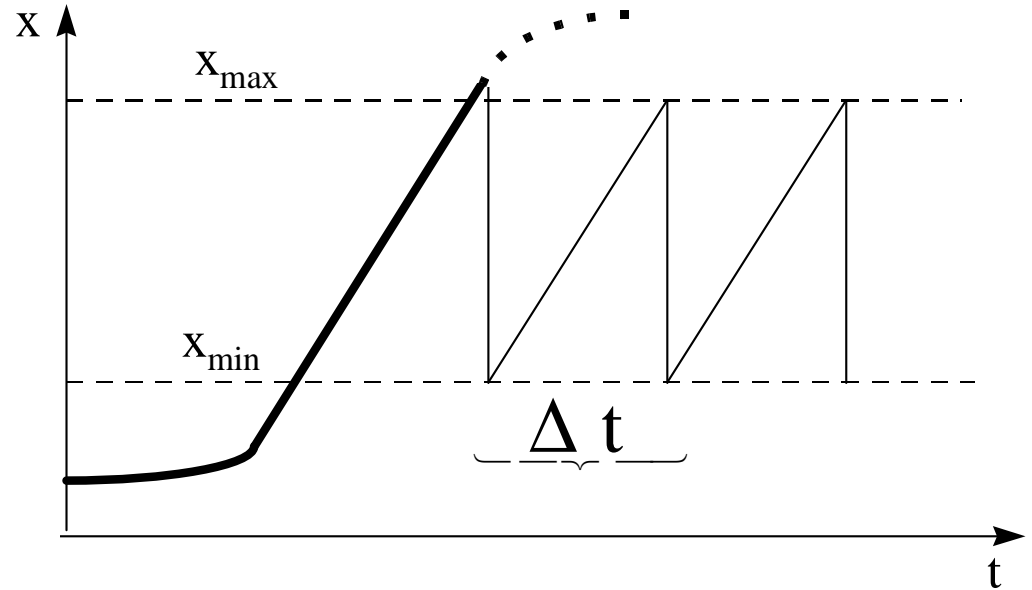
# OTHER CULTIVATION METHODS

## Semicontinuous fermentation

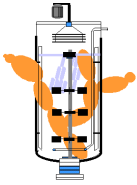
$$X_{\max} = X_{\min} e^{\mu \Delta t} \quad \text{vagy} \quad \ln \frac{X_{\max}}{X_{\min}} = \mu \Delta t$$

$$D = \frac{\alpha V}{\Delta t} \frac{1}{V} = \frac{\alpha}{\Delta t} = \frac{\alpha \mu_{\max}}{\ln \frac{X_{\max}}{X_{\min}}}$$

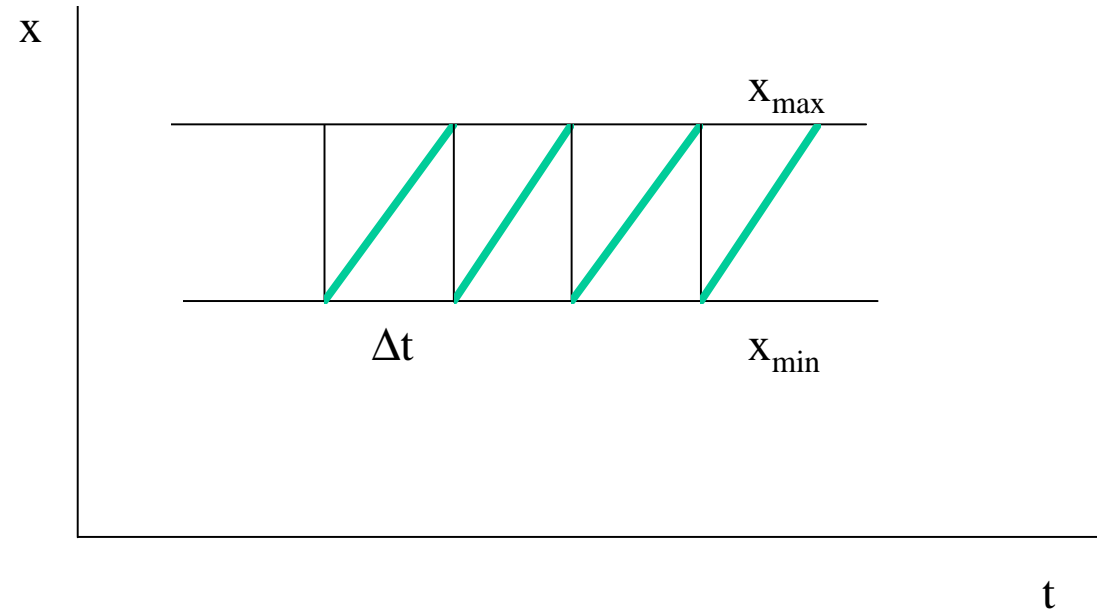
$$J = D \cdot X = \frac{\alpha \mu_{\max}}{\ln \frac{X_{\max}}{X_{\min}}} X_{\max}$$



$\alpha \cdot V$  volume taken off



## Other... TURBIDOSTAT



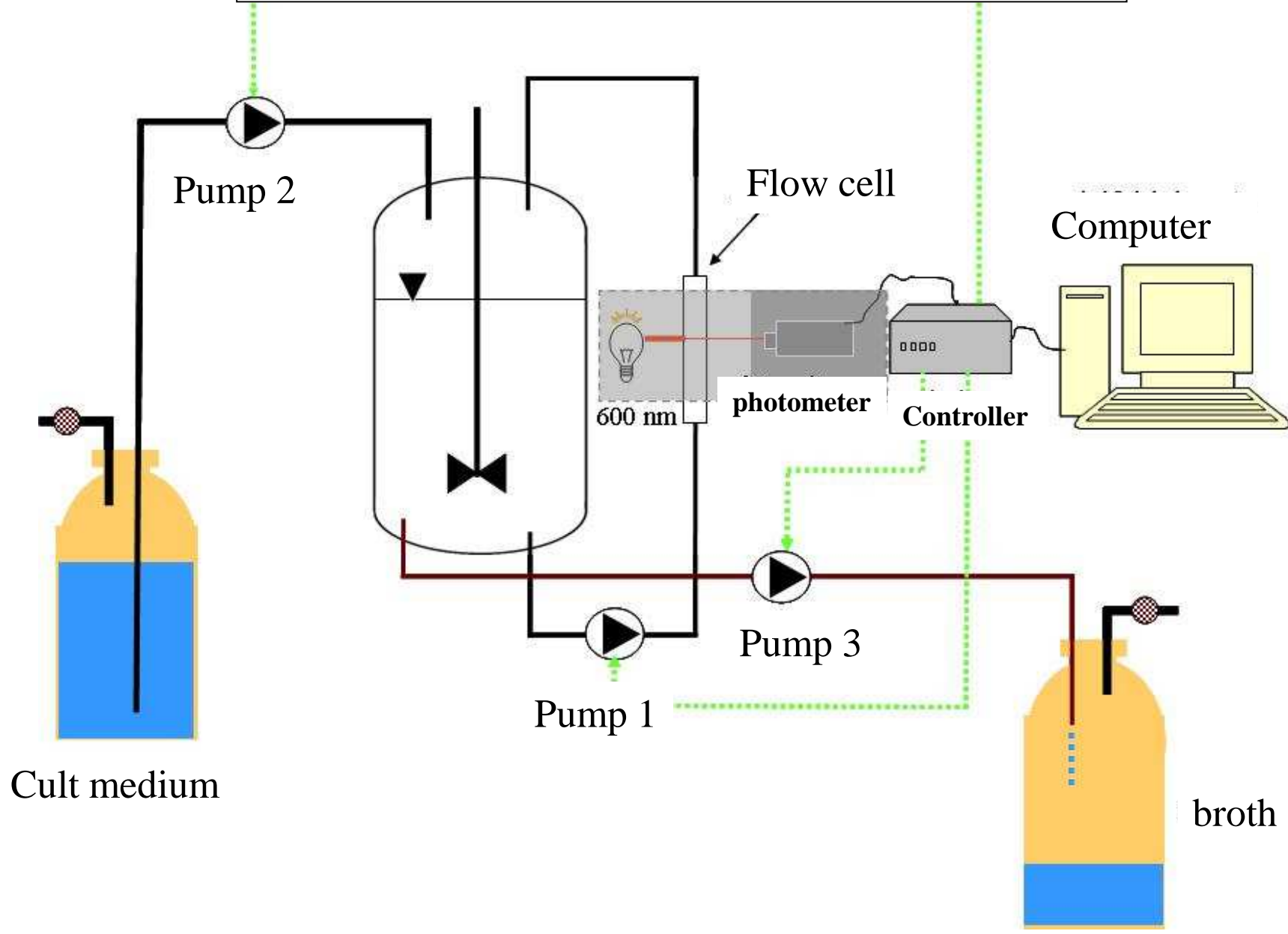
$$\frac{dx}{dt} \cong \frac{\Delta x}{\Delta t} = \frac{x_{\max} - x_{\min}}{\Delta t}$$

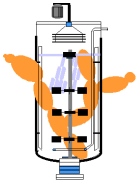
$\mu = \mu_{\max}$  is possible!!!

$$\mu = \frac{1}{x} \frac{dx}{dt} \cong \frac{1}{x} \frac{\Delta x}{\Delta t} = \frac{2}{x_{\max} + x_{\min}} \frac{x_{\max} - x_{\min}}{\Delta t}$$



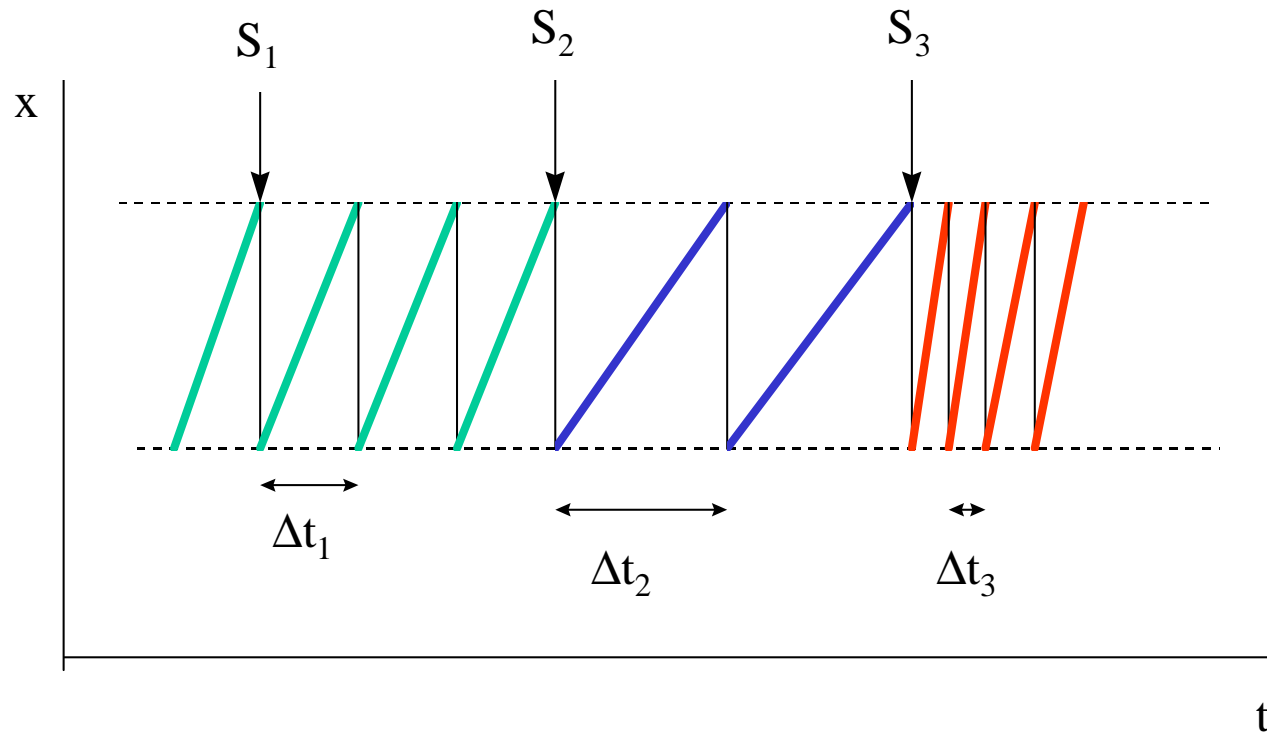
Other....: TURBIDOSTAT

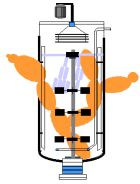




Other.... TURBIDOSTAT

Application for research: optimization





## Other.....fed batch fermentation

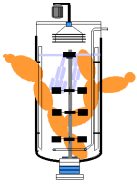
### Fed batch fermentation

Continuation of the declining phase, constant, variable or periodic addition of fresh cult. medium, **no broth removal.**

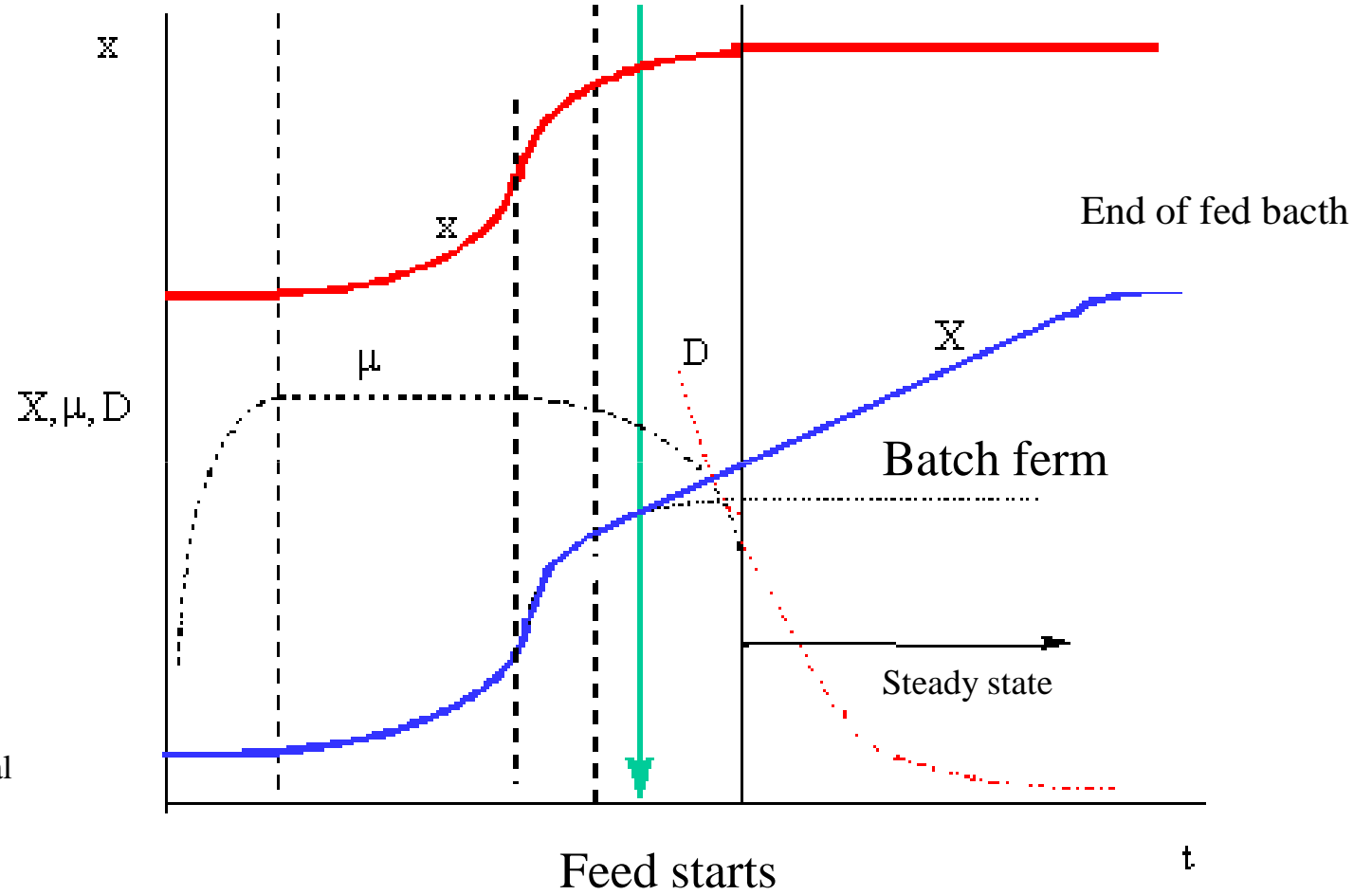
- \*keeping low, constant S concentration (Baker's yeast: glucose repression, Crabtree effect),
- \*high constant S concentration (citric acid fermentation)
- \*precursor continuous addition (penicillin: phenyl-acetic-acid, )

**pH control!!**

Varying volume,  $f(t)$



Other.....fed batch fermentation



$$V_{\text{start}} \cong 0,5-0,6 V_{\text{total}}$$

$$V_{\text{end}} \cong 0,7-0,85 V_{\text{total}}$$