HYPOTHESIS TESTS

We are interested in the population, but the sample is in our hands.

Some assumption is made on the population (e.g. the value of μ and/or σ), and this assumption is accepted or rejected based on the data.

May the data come from a distribution …? E.g. $\mu=\mu_0$?

$H_0 : \mu = \mu_0$

Null hypothesis

$H_1 : \mu \neq \mu_0$

Alternative hypothesis

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z-test

$\mathrm{H}_0 : \mu = \mu_0$ $\mathrm{H}_1 : \mu \neq \mu_0$

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ ⟶ $z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ test statistic

If H_0 is true, z_0=z $P\left(-z_{\alpha/2} < \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \leq z_{\alpha/2} \right) = 1 - \alpha$

If z_0 takes its value in the usual range, accepted.

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z-test

$\mathrm{H}_0 : \mu = \mu_0$ $\mathrm{H}_1 : \mu \neq \mu_0$

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ ⟶ $z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ test statistic

$P\left(-z_{\alpha/2} < \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \leq z_{\alpha/2} \right) = 1 - \alpha$

α is the significance level

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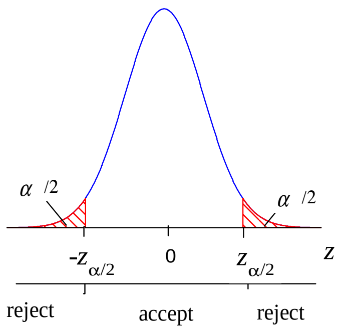
Region of acceptance



$$P\left(-z_{a/2} < z_0 \le z_{a/2} \middle| \mathrm{H}_0\right) = 1-\alpha$$

$$z_0 = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$$

$$P\left(-z_{a/2} < \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \le z_{a/2} \middle| \mathrm{H}_0\right) = 1-\alpha$$

$$P\left(\mu_0 - z_{\alpha/2}\sigma/\sqrt{n} < \bar{x} < \mu_0 + z_{\alpha/2}\sigma/\sqrt{n}\right) = 1-\alpha$$

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Confidence interval and hypothesis test

$$P\left(-z_{a/2} < \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \le z_{a/2} \middle| \mathrm{H}_0\right) = 1-\alpha$$

$$P\left(\bar{x} - z_{\alpha/2}\sigma/\sqrt{n} < \mu_0 \le \bar{x} + z_{\alpha/2}\sigma/\sqrt{n}\right) = 1-\alpha$$

confidence interval for μ:

$$P\left(\bar{x} - z_{\alpha/2}\sigma/\sqrt{n} < \mu \le \bar{x} + z_{\alpha/2}\sigma/\sqrt{n}\right) = 1-\alpha$$

If the confidence interval contains the hypothesised μ_0 value, H_0 is accepted.

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Example 1

The mass of an object is measured with 4 repeated measurements.
The sample mean is 5.0125 g.
From historical data the variance is known as $\sigma^2 = 10^{-4}$ g^2.

May we believe (based on the data) that the expected value (the true mass of the object if the balance is unbiased) is 5.0000 g?

$\mathrm{H}_0 : \mu = \mu_0$ $\qquad$ $\mathrm{H}_1 : \mu \neq \mu_0$

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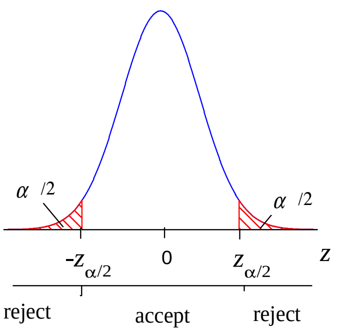
E.g in case of $\mathrm{H}_1 : \mu \neq \mu_0$

$$P\left(-z_{a/2} < \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \leq z_{a/2} \right) = 1 - \alpha$$



Is the value of the test statistic in the region of acceptance?

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$$H_0 : \mu = \mu_0 = 5.0000, \quad H_1 : \mu \neq \mu_0 = 5.0000$$

$$\bar{x} = 5.0125, \quad \sigma^2 = 10^{-4}, \quad n = 4, \quad \alpha = 0.05$$

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} =$$

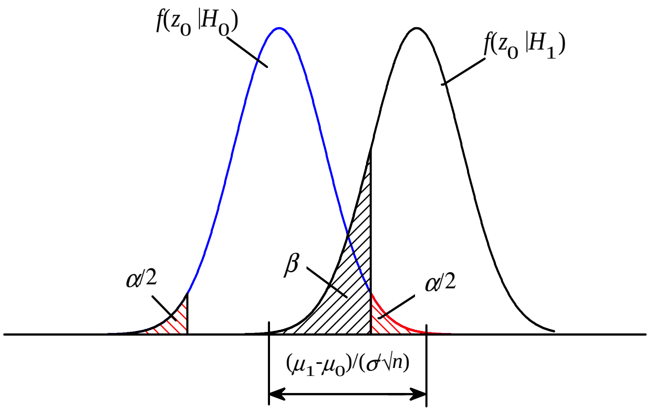
$$z_{\alpha/2} =$$

Error of first and second kind

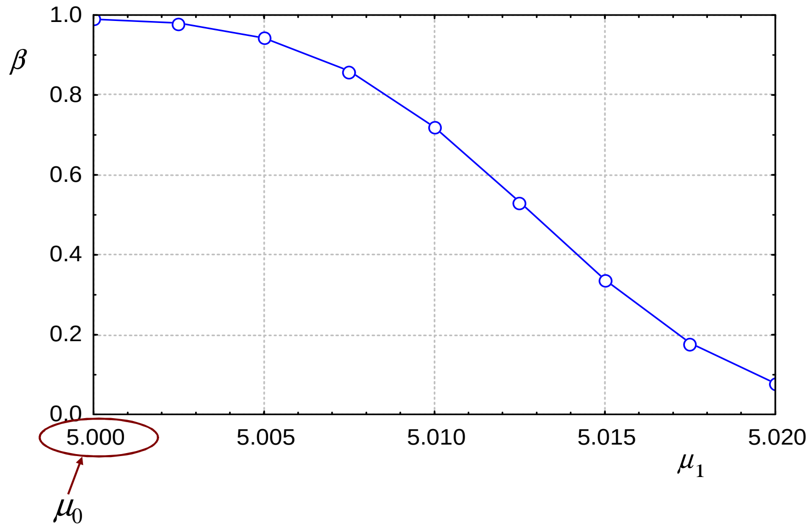
	Decision	
	The H_0 hypothesis is	
	accepted	rejected
H_0 is true	Proper decision	Error of first kind (α)
H_0 is false	Error of second kind (β)	Proper decision

"fail to reject"

Probability of committing an error of second kind



OC (operating characteristic) curve



One-sample t test

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \qquad t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

$$\mathrm{H}_0 : \mu = \mu_0 \qquad \mathrm{H}_1 : \mu \neq \mu_0$$

$$P\left(-t_{a/2} < t_0 \leq t_{a/2}\right) = P\left(-t_{a/2} < \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \leq t_{a/2} \right) = 1 - \alpha$$

$$P\left(\bar{x} - t_{a/2}\, s/\sqrt{n} < \mu_0 \leq \bar{x} + t_{a/2}\, s/\sqrt{n}\right) = 1 - \alpha$$

CI contains the hypothesised μ_0 value, accepted

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Example 2
Checking the bias of a gauge

$$\mathrm{H}_0 : E(x) = x_{ref} \qquad \mathrm{H}_1 : E(x) \neq \mu_0 = x_{ref}$$

x_{ref}=6.0 (standard)

$$t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

i	x_i	$x_i - x_{ref}$
1	5.8	-0.2
2	5.7	-0.3
3	5.9	-0.1
4	5.9	-0.1
5	6.0	0.0
6	6.1	0.1
7	6.0	0.0
8	6.1	0.0
9	6.4	0.4
10	6.3	0.3
11	6.0	0.0
12	6.1	0.1
13	6.2	0.2
14	5.6	-0.4
15	6.0	0.0

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CI contains the hypothesised μ_0=6.0 value, accepted

Test of means against reference constant (value) (example 12)

Variable	Mean	Std.Dv.	N	Std.Err.	Confidence -95.000%	Confidence +95.000%	Reference Constant	t-value	df	p
Var1	6.006667	0.212020	15	0.054743	5.889254	6.124079	6.000000	0.121781	14	0.904804

p is the probability of obtaining this or more extreme result if H_0 is true (probability of error of first kind)
Std. Err.: standard error of mean

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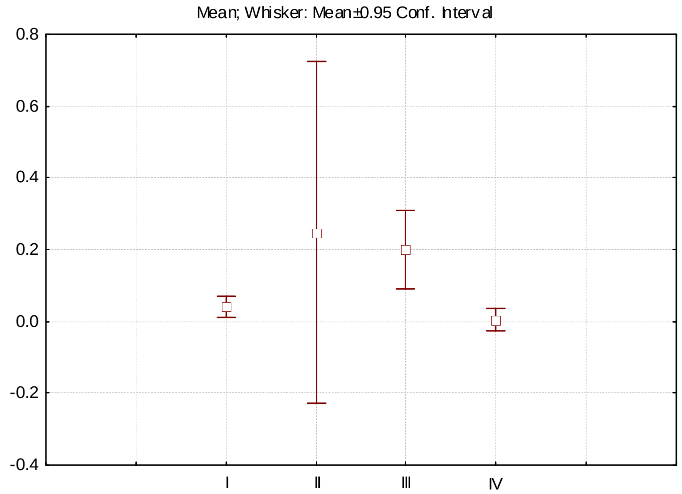
J. H. Steiger, R.T. Fouladi: Noncentrality Interval Estimation and the Evaluation of Statistical Models, Chapter 9 in: L.L. Harlow, S.A. Mulaik, J.H. Steiger: What if there were no significance tests? Mahwah, NJ: Erlbaum (1997)



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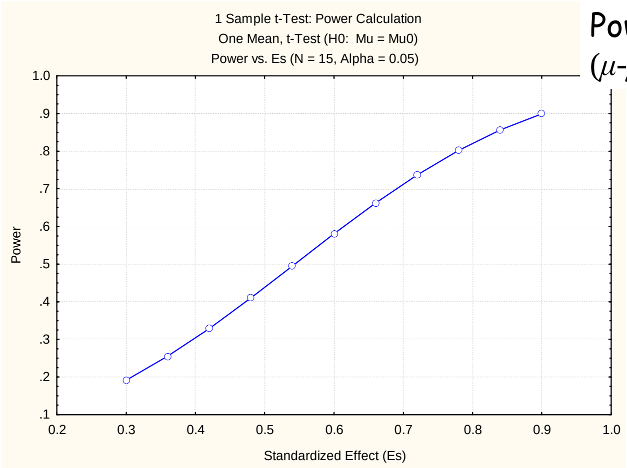
Power, statistically significant difference

Power=1-β certainty of detection



Power depends on:
$(\mu-\mu_0)$, σ , n, α

standardized effect:

$$Es = \frac{\mu - \mu_0}{\sigma}$$

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The sample size (n=15) and error of first kind is fixed (α=0.05) , σ = 0.212.
What difference $(\mu-\mu_0)$ can be detected with 90% probability (β=0.1)?

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