Attributes control charts

- charts for defectives (*np* and *p*)
  based on Binomial distribution
- charts for occurrences (defects) (*c* and *u*)
  based on Poisson distribution

Control charts for count of defectives: *np chart*

p is the proportion of defectives in the population (process), its estimate is the proportion of defectives in the sample :

$$\hat{p} = \frac{x}{n}$$

Binomial distribution:

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\mu_x = E(x) = np$$

$$\sigma_x^2 = Var(x) = np(1-p)$$

$$\mu_{x/n} = E\left(\frac{x}{n}\right) = p$$

$$\sigma_{x/n}^2 = Var\left(\frac{x}{n}\right) = \frac{p(1-p)}{n}$$

The parameters of the *np* chart according to the ±3σ rule

$$E(x) = np \qquad CL_{np} = n\bar{p}$$

$$Var(x) = np(1-p) \qquad UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

If *LCL* is <0, set to zero.

$\bar{p}$ is the average proportion of defectives

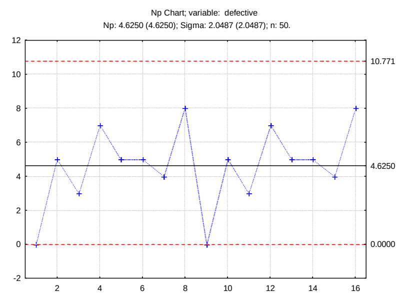
Example 1

50 pieces are drawn in each half an hour from a process.
Number of defectives:

time	8:00	8:30	9:00	9:30	10:00	10:30	11:00	11:30
$D(np)$	0	5	3	7	5	5	4	8

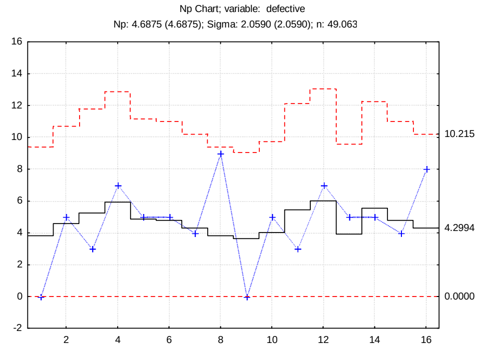
time	12:00	12:30	13:00	13:30	14:00	14:30	15:00	15:30
$D(np)$	0	5	3	7	5	5	4	8

Prepare an *np* chart assuming the situation of a Phase I study!



Why do we have a single chart?

np- chart with changing sample size



Control chart for proportion of defectives: *p chart*

$$\hat{p} = \frac{D}{n} \qquad E(\hat{p}) = p \qquad Var(\hat{p}) = \frac{p(1-p)}{n}$$

The parameters according to the ±3σ rule:

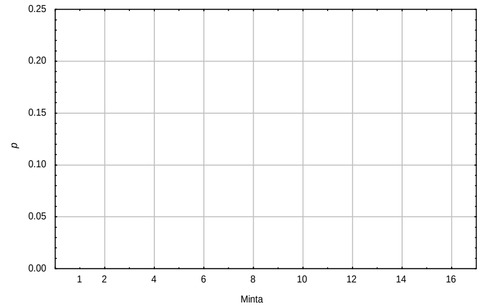
$$CL_p = \bar{p}$$

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

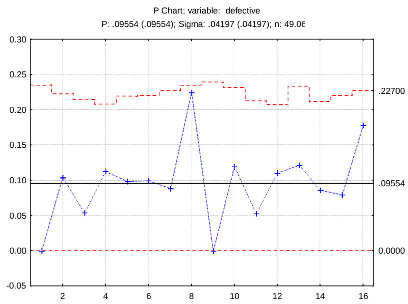
time	D	*n*
8:00	0	40
8:30	5	48
9:00	3	55
9:30	7	62
10:00	5	51
10:30	5	50
11:00	4	45
11:30	9	40
12:00	0	38
12:30	5	42
13:00	3	57
13:30	7	63
14:00	5	41
14:30	5	58
15:00	4	50
15:30	8	45

Example 2

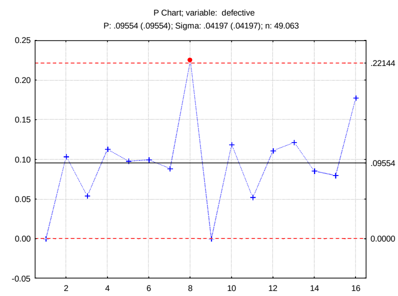
Prepare a *p* chart the data for!



using actual sizes of subgroups



p chart with average control limits



Control charts for occurrence of defects: *c* chart

Poisson distribution

for modelling rare events

x is the number of occurrences, „from among how many” is not defined

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

Expected value and variance: $E(x) = Var(x) = \lambda$

λ is the expected number of occurrences in a unit

Defect charts: c chart

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad \lambda = np$$

$$E(x) = \lambda \qquad Var(x) = \lambda$$

The *x* average number of defects obtained in Phase I is the estimate of the λ parameter :

$$\bar{c} = \frac{\sum_i^m c_i}{m}$$

c_i # of defects found in sample *i*
m # of samples checked

In Phase II (on-going control) the parameters of the charts using the ±3σ rule:

$$CL_c = \bar{c}$$

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

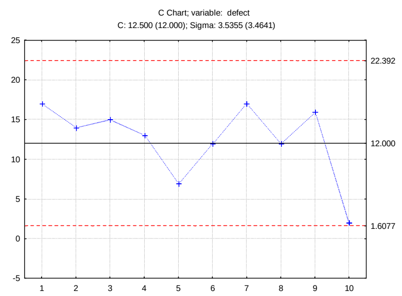
$\bar{c}$ is the value obtained in Phase I.

sample	# defects
1	17
2	14
3	15
4	13
5	7
6	12
7	17
8	12
9	16
10	2

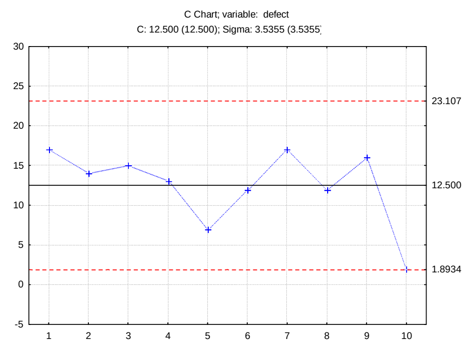
Example 3

The average number of painting defects on car doors manufactured is 2. The doors are sampled for checking, 6 doors are considered as a sample.
Prepare a *c* chart for checking stability of the process!

Phase I or Phase II?



Considering as Phase I study:

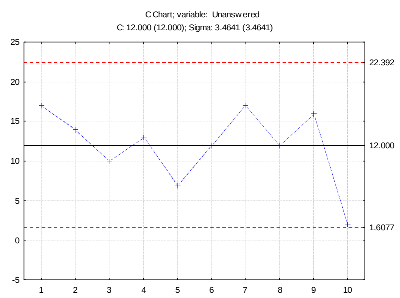


Example 4

The average number of unanswered calls in a call center is 2 per hour (from earlier studies). Each week 6 hours are checked and considered as 1 sample.
Prepare a *c* chart for checking stability of the process!

week	# unanswered
1	17
2	14
3	10
4	13
5	7
6	12
7	17
8	12
9	16
10	2

Phase I or Phase II?



Control charts for occurrence of defects: *u* chart

The size of the sample may not be constant

E.g.

the car doors may not be of the same type,
the number of pieces on days are different
the complexity of bills may be different,
the number of calls on different days is different

Comparison of variables and attributes control charts

variables: continuous random variable
attributes: discrete random variable
The variables charts:

- offer more information, more sensitive to changes, the signal the special causes (e.g. shift) before defectives are manufactured, since the specification limits are not necessarily reached when control limits are exceeded.
- require much smaller sample size, but the measurement is usually more expensive then deciding on attributes, and the former is not always applicable.

variables data
- data collected in groups: X-bar/R
- individual data: I/MR, X/MR

attribute data
- nonconforming items
  - sample size is constant: np or p
  - sample size is changing: np or p
- defects
  - sample size is constant: c
  - sample size is changing: u