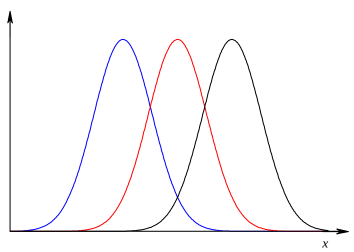
Rehearsal

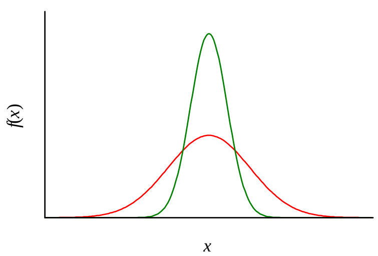
1

The most important continuous distribution: Gauss (normal) distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2 \right]$$

Two parameters: μ and σ^2





μ is different

σ is different

2

Expected value and variance:

$$E(x)=\mu \qquad Var(x)=\sigma^2$$

Short notation:

$$N(\mu,\sigma^2) \qquad \text{e.g.} \qquad N(0,1)$$

Standardisation:

$$z=\frac{x-\mu}{\sigma} \qquad f(z)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)$$

$$\mu=E(z)=0 \qquad \sigma^2=Var(z)=1$$

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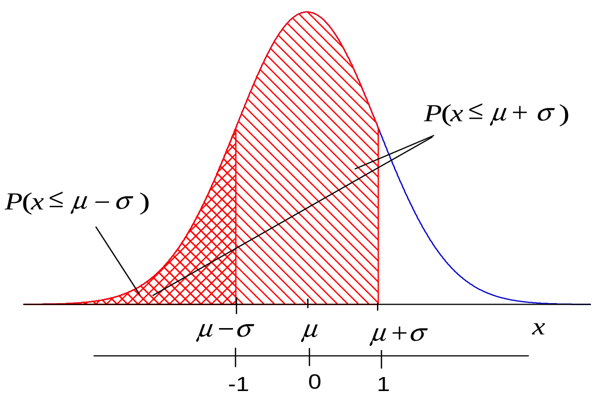
What is the probability of finding the x Gauss d. random variable in the $(\mu-\sigma,\mu+\sigma)$ range?

$$P(\mu-\sigma<x\le\mu+\sigma)=F(\mu+\sigma)-F(\mu-\sigma)$$



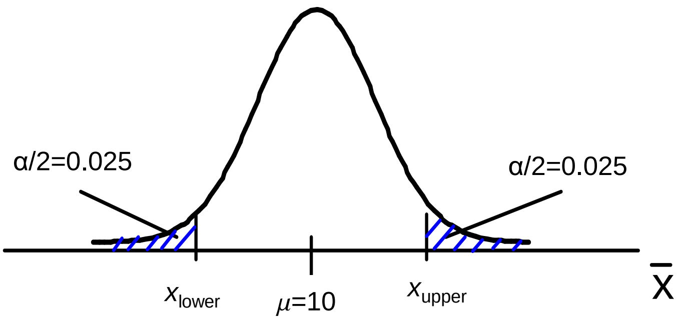
4

$$z_{lower} = \frac{\mu - \sigma - \mu}{\sigma} = -1 \qquad z_{upper} = \frac{\mu + \sigma - \mu}{\sigma} = 1$$

Width of the interval	$\pm\sigma$	$\pm 2\sigma$	$\pm 3\sigma$
P	0.68268	0.9545	0.9973

The variance of a measurement is σ^2 =0.25g^2. The measurement is unbiased. We measure an object, its weight is 10 g. In which range will be the outcome of the measurement with 95% probability?

1-α=0.95% α=0.05



The variance of a measurement is σ^2 =0.25g^2. The measurement is unbiased. We measure an object, its weight is 10 g. In which range will be the outcome of the measurement with 95% probability?

This is the question:

$$P\left(x_{lower} < x \le x_{upper}\right) = 0.95$$

This is what we know from the distribution function:

$$P\left(-z_{\alpha/2} < z \le z_{\alpha/2}\right) = 0.95$$

Connection: $z = \dfrac{x-\mu}{\sigma}$

$$P\left(-z_{\alpha/2} < \frac{x-\mu}{\sigma} \le z_{\alpha/2}\right) = 0.95$$

$$P\left(\overbrace{\mu - z_{\alpha/2}\,\sigma/\sqrt{n}}^{x_{lower}} < x \le \overbrace{\mu + z_{\alpha/2}\,\sigma/\sqrt{n}}^{x_{upper}}\right) = 1-\alpha$$

$$P\left(10 - 1.96\cdot 0.5 < x \le 10 + 1.96\cdot 0.5\right) = 0.95$$

7

Central limit theorem

The mean of sample elements taken from any distribution approximately follows Gauss d. around the expected value of the original d. with variance σ^2/n.

The approximation steadily improves as the number of observations increases.

$$\bar{x} \sim N(\mu, \sigma^2/N)$$

Sum as well $\sum_{i=1}^{N} x_i \sim N(N\mu, N\sigma^2)$

Based on the Central Limit Theorem: $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

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Calculate the 95% probability interval for the mean of a n = 5 sample taken from a population of μ=10 and σ^2=0.25 !

This is the question:

$$P\left(\overline{x}_{lower} < \overline{x} \leq \overline{x}_{upper}\right) = 0.95$$

This is what we know from the distribution function:

$$P\left(-z_{\alpha/2} < z \leq z_{\alpha/2}\right) = 0.95$$

Connection: $z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}}$

$$P\left(-z_{\alpha/2} < \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \leq z_{\alpha/2}\right) = 0.95$$

$$P\left(\overbrace{\mu - z_{\alpha/2}\,\sigma/\sqrt{n}}^{\overline{x}_{lower}} < \overline{x} \leq \overbrace{\mu + z_{\alpha/2}\,\sigma/\sqrt{n}}^{\overline{x}_{upper}}\right) = 1 - \alpha$$

$$P\left(10 - 1.96 \cdot 0.5/\sqrt{5} < \overline{x} \leq 10 + 1.96 \cdot 0.5/\sqrt{5}\right) = 0.95$$

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The size of the produced parts (from a particular process) follows normal distribution. The expected value is 10, the variance is 0.25. The lower specification limit is 8.5 cm. What is the ratio of noncomforming parts from this process?

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