VARIABLES CONTROL CHARTS

Variables control charts

Example 1

The assumed expected value of the mass of packages produced by an automatic machine is 250 g, the known variance of the process is 1 g^2.
The mean of the sample of 5 elements taken from the process is:

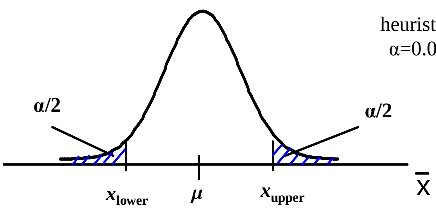
$$\overline{x} = 249.6 \text{ g}$$

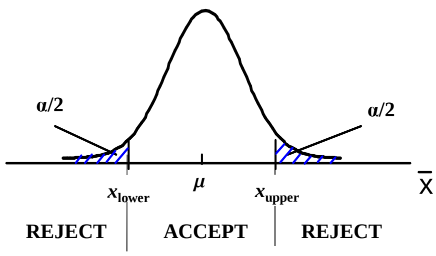
Do we believe that the expected value of the mass of packages is 250 g?

$$P(\mu - z_{\alpha/2}\,\sigma/\sqrt{n} < \overline{x} < \mu + z_{\alpha/2}\,\sigma/\sqrt{n}) = 1-\alpha$$

α: probability of having the average out of this range.

heuristic:
α=0.05





If the average is in the accept/reject region we accept/reject, that the expected value is 250g.

The region of acceptance:

$$\mu - z_{\alpha/2}\,\sigma/\sqrt{n} < \overline{x} < \mu + z_{\alpha/2}\,\sigma/\sqrt{n}$$

$$UCL = \overline{x}_{\text{upper}} = \mu + z_{\alpha/2}\sigma/\sqrt{n} =$$ $z_{\alpha/2} =$

$$LCL = \overline{x}_{\text{lower}} = \mu - z_{\alpha/2}\sigma/\sqrt{n} =$$

Decision:

The region of acceptance:

$$\mu - z_{\alpha/2}\,\sigma/\sqrt{n} < \overline{x} < \mu + z_{\alpha/2}\,\sigma/\sqrt{n}$$

Take samples (subgroup) time to time and plot their mean as a function of time!

→ control chart

- in statistical control: continue
- out of control: stop the process

The intervention is usually expensive (the manufacturing line is stopped), thus the chance for false alarm is to be diminished:

$z_{\alpha/2}$ =3 (the so called $\pm 3\sigma$ limit),

then α=0.0027, that is the chance for erroneous decision is about three from among one thousand.

$$\mu_0 - 3\sigma/\sqrt{n} < \bar{x} < \mu_0 + 3\sigma/\sqrt{n}$$

LCL ***UCL***

The region of acceptance:

$$\mu_0 - 3\sigma/\sqrt{n} < \bar{x} < \mu_0 + 3\sigma/\sqrt{n}$$

Problem 1

μ_0 and σ are not known (we do not know the reference to which the process is to be compared)

→ estimation from a large sample

Problem 2

We may not be sure if the process used for estimating μ and σ is in control

→ check using control chart

Phase I: establishing stability and control limits

Phase II: on-going control using the previously established control limits

The X-bar - Range chart

n (typically *n*=3 - 5) samples are taken from the process time to time. The mean and the range of the sample is computed:

$$R = |x_{max} - x_{min}| \qquad \bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$$

An R_i range and $\bar{x}_i$ mean is found for the sample *i*.

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \quad \text{where} \quad \bar{R} = \frac{1}{m}\sum_i R_i$$

Construction of the X-bar chart

Phase I

$CL_{\bar{x}} = \bar{\bar{x}} = \frac{1}{m}\sum_i \bar{x}_i$ (*m* is the number of samples, $\bar{x}_i$ is the mean of the *i*-th sample)

$UCL_{\bar{x}} = \bar{\bar{x}} + \frac{3\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} + A_2\bar{R}$ (upper control limit)

$LCL_{\bar{x}} = \bar{\bar{x}} - \frac{3\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} - A_2\bar{R}$ (lower control limit)

Phase II (on-going control)

$\bar{\bar{x}}$ and $\bar{R}$

from Phase I, that is the center line and control limits are given

Construction of the range (R) chart

$H_0 : Var(x) = \sigma_0^2$

Phase I

$$CL_R = \bar{R} = \frac{1}{m}\sum_i R_i \qquad \hat{\sigma}_R = d_3\hat{\sigma} = \frac{d_3\bar{R}}{d_2} = \frac{(D_4-1)\bar{R}}{3}$$

The control limits for the $\pm 3\sigma$ rule:

$$UCL_R = \bar{R} + 3\hat{\sigma}_R = \bar{R} + 3\frac{d_3\bar{R}}{d_2} = \bar{R}\left(1+3\frac{d_3}{d_2}\right) = D_4\bar{R}$$

$$LCL_R = \bar{R} - 3\hat{\sigma}_R = \bar{R} - 3\frac{d_3\bar{R}}{d_2} = \bar{R}\left(1-3\frac{d_3}{d_2}\right) = D_3\bar{R}$$

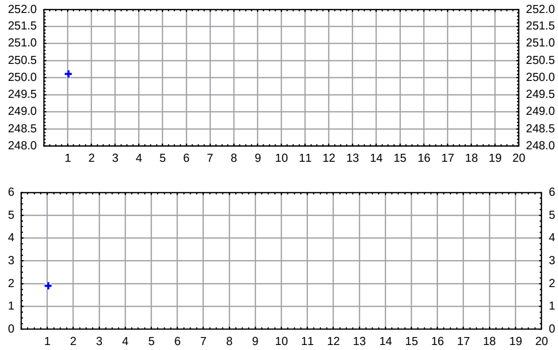
If negative value is obtained for *LCL*, it is to be set as zero

n	d_2	d_3	c_4	A_2	A_3	B_3	B_4	D_3	D_4
2	1.128	0.853	0.7979	1.880	2.659	0	3.267	0	3.267
3	1.693	0.886	0.8862	1.023	1.954	0	2.568	0	2.574
4	2.059	0.880	0.9213	0.729	1.628	0	2.266	0	2.282
5	2.326	0.864	0.9400	0.577	1.427	0	2.089	0	2.114

Example 2

Prepare an X-bar/R chart using the data in the table!

i	**measured sample elements**					**mean**	***R***
1	251.25	249.67	250.15	250.22	249.30	250.118	1.950
2	247.56	249.84	251.04	249.47	250.25		
3	251.47	250.23	250.07	250.12	250.37		
4	249.35	249.77	249.29	250.92	250.44	249.954	1.630
5	249.09	251.09	248.14	248.51	250.90	249.546	2.950
6	251.59	248.13	250.06	248.92	252.09	250.158	3.960
7	250.61	249.55	249.23	249.61	251.39	250.078	2.160
8	249.95	247.74	249.40	248.88	249.16	249.026	2.210
9	247.74	249.42	249.59	251.59	250.36	249.740	3.850
10	247.89	250.65	249.61	249.08	248.72	249.190	2.760
11	249.26	250.08	251.22	250.08	250.26	250.180	1.960
12	249.83	249.46	248.83	251.56	249.16	249.768	2.730
13	250.36	250.10	251.68	250.36	248.78	250.256	2.900
14	250.71	250.26	250.18	249.47	250.72	250.268	1.250
15	250.50	252.36	251.52	249.91	250.75	251.008	2.450
16	250.11	250.87	249.31	249.93	249.63	249.970	1.560
17	248.81	249.65	248.08	250.57	251.48	249.718	3.400
18	249.90	249.81	250.59	250.38	250.74	250.284	0.930
19	250.88	249.79	249.85	250.11	250.61	250.248	1.090
20	249.27	248.61	250.64	249.43	249.60	249.510	2.030
mean						249.955	2.333

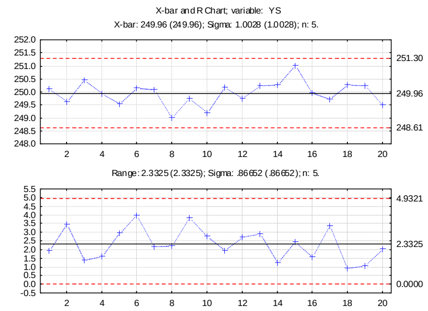


Example 3

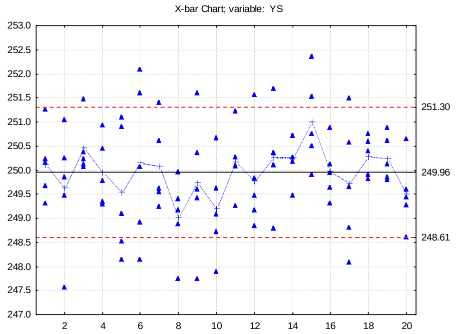
Prepare an X-bar/R chart using the YS column of the cpdata1.sta data file!

Phase I or Phase II?

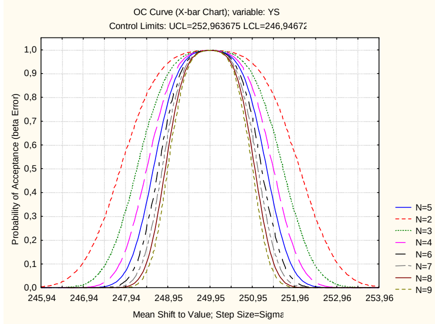
Open cpdata1.sta
Statistics>Industrial Statistics>Quality Control Charts
X-bar & R chart for variables
Variables: YS, Sample



The control limits on the X-bar chart refer to the mean, not to single measurement values!

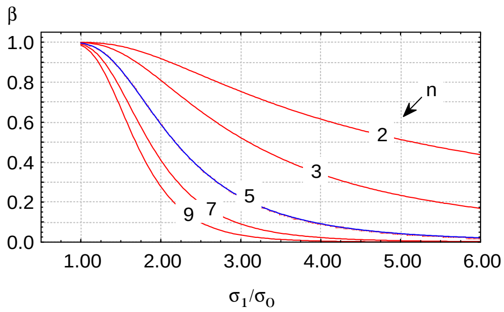


Operating Characteristic (OC) curve for the X-bar chart (α=0.0027)



The true mean is on the horizontal axis

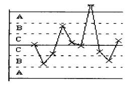
Operating Characteristic (OC) curve for the R chart (±3σ, that is α=0.0027?)



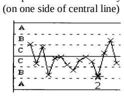
The Western Electric algorithmic rules (run tests)

Western Electric rules (runs test)

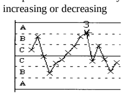
1. One point beyond Zone A



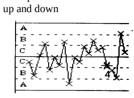
2. 9 points in Zone C or beyond (on one side of central line)



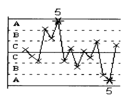
3. 6 points in a row steadily increasing or decreasing



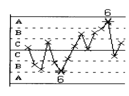
4. 14 points in a row alternating up and down



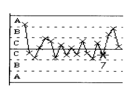
5. 2 out of 3 points in a row in Zone A or beyond



6. 4 out of 5 points in a row in Zone B or beyond



7. 15 points in a row in Zone C (above and below the center line)



8. 8 points in a row in Zone B, A, or beyond, on either side of the center line (without points in Zone C)